The Mutual Inductance Effective Permeability and its Application

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Abstract: Self inductance effective permeability is used to simplify the magnetic equivalent circuit of an inductor with magnetic core. It represents how the high magnetic permeability material (e.g. ferrite core), which takes place only in some area around the coil winding, influences the electrical characteristics of the inductor. Similarly the mutual inductance effective permeability can be introduced, which represents how the magnetic core of the inductor influences the surrounding external magnetic field and the flux density inside the magnetic core. The mutual inductance effective permeability can be used to calculate the sensitivity of transponder coils or antenna coils in wireless communication or RFID applications. It is also capable of comparing different coil constructions, or can be used for optimization of a design to achieve the desired sensitivity. The calculation should be carried out by numerical methods because of the inhomogeneous medium. Due to the simplicity of the problem the hardware and computation time requirements are relative low, which allows the handling of very complex geometries without simplification. The accuracy of the method was tested on market available Transponder coils, and the computed results were in good agreement with the measured sensitivity.

Keywords: effective permeability, sensitivity, transponder coil, LF

1. Introduction

1.1. Antenna coils in inductively coupled systems

Low frequency inductive coupling is widely used operating principle in wireless communication, wireless identification or energy transfer applications, e.g. RFID (Radio Frequency Identification), NFC (Near Field Communication). The operating principle of these applications is the near field inductive coupling where two or more coils are coupled weakly by the mutual magnetic flux. The key component of these applications is the antenna coils or transponder coils that connect the transmitter with the reader unit. The coil parameters should be considered and estimated in the early phase of the inductor development process, so thus accurate and efficient methods are required for determining the coil parameters. The currently available techniques mainly focus on parameters related to the self inductance, e.g. impedance, frequency dependent resistance, inductance, quality factor. In inductively coupled systems not only the self inductance related parameters but the mutual inductance related parameters are also very important, such as coupling coefficient and sensitivity. The quality of the coupling between two coils can be represented by the coupling coefficient, but it refers not only one component, but both primary and secondary antenna coils which are coupled. Since the transponder or tag antenna coil should be independent from the reader coil, the sensitivity is the parameter which is capable to give information about the performance of the transponder coil design without knowing any parameter of the reader coil. The aim of the paper is to give a brief summary about the sensitivity parameter and its definition, to introduce the mutual inductance effective permeability and its calculation, and to present a calculation method using the mutual inductance effective permeability for estimating the sensitivity parameter of transponder and antenna coils.

1.2. Sensitivity parameter

The Sensitivity parameter gives the most information about the performance of the Transponder coil, showing how sensitive the coil is to the changing external magnetic field.

The sensitivity is defined as the quotient of the induced voltage across the inductor due to changing magnetic field and the intensity of the magnetic field, which induces the voltage [1,2].

$$S = \frac{U_i}{H} \text{ or } S = \frac{U_i}{B}, \tag{1}$$

where U_i is the induced voltage across the inductor, and H and B is the magnetic field intensity, and the magnetic flux density.

The measurement of the sensitivity can be performed with the help of a device, which is capable of creating uniform (constant in magnitude and in orientation as well) electromagnetic field. These devices are the Helmholtz coil and the Maxwell coil. A typical sensitivity test setup consists of a signal generator, a Helmholtz coil and an oscilloscope [3].

The inductor is measured as a single part, not as a resonant circuit. There is not any attached component like tuning capacitor or damping resistor, and the parasitic components of the inductors (e.g. stray capacitance between turns and layers) are neglected as well. The excitation signal is a sine wave in the low frequency range, which is supplied by the signal generator, and feeds the Helmholtz coil, which generates the changing electromagnetic field. The induced voltage on the transponder coil is measured by an oscilloscope, and the sensitivity can be calculated using the formula (1). This kind of measurement results one scalar parameter, which allows the comparison of different transponder coils.

1.3. Self inductance effective permeability

Self inductance effective permeability is used to simplify the magnetic equivalent circuit of an inductor. The inductance calculation of coils in inhomogeneous medium is difficult and analytical formulas are not applicable and available due to the wide variety of the core shapes and the wide range of permeability of the used magnetic materials.

The air gap in most cases is so large that the flux fringing can not be neglected. The ferrite core of the inductor obviously has much higher relative permeability than 1, but the overall average permeability of the complete magnetic circuit is lower than the permeability of the ferrite itself [4].

The average overall permeability of the inductor is the self inductance effective permeability. It represents how the high permeability material, which takes place only in some area around the coil, influences the inductance of the inductor. In other words, the self inductance effective permeability represents the permeability of an equivalent homogeneous medium which results the same inductance value with the same coil geometry. The effective permeability is a useful indicator parameter during the inductor development, which helps in material selection, and design evaluation. Calculation of the self inductance effective permeability can be performed by the reluctance model method or can be computed by numerical methods. The self inductance effective permeability is applicable only for determining self inductance related parameters of an inductor.

2. Mutual Inductance Effective Permeability

2.1. Definition of mutual inductance effective permeability

Analogically to the Self Inductance Effective Permeability, the Mutual Inductance Effective Permeability can be introduced. It represents how the magnetic core influences the magnetic flux density inside the coil, when the inductor is placed in an external magnetic field. Due to the shape and the magnetic permeability of the magnetic core, it will have lower magnetic reluctance, which changing the external magnetic field around the core so that the flux will increase inside the core. The mutual inductance effective permeability in most cases shows anisotropic property due to the asymmetric geometry. The mutual inductance effective permeability can be used to calculate the sensitivity parameter, and helps to evaluate and compare different inductor constructions.

The calculation consists of three main steps. The first one is the definition (value and direction) of the external homogenous magnetic field intensity H_E, where the inductor shall be placed [5]. The inductor (magnetic core) shall be aligned in the desired position so that the axis of the coil is parallel with the direction of the magnetic field. In the second step the average flux density B_m shall be calculated on the cross sectional area on the middle (or where the coil winding takes place on the core) of the inductor. In the third step the quotient of the average flux density (calculated in the second step) and the initial external flux density shall be calculated,

$$\mu_{M\_eff} = \frac{B_m}{\mu_0 H_E}. \tag{2}$$

The quotient calculated in the third step μ_{M_eff} is the mutual inductance effective permeability.

The computation of the second step is performed by numerical methods, e.g. Finite Element Method.

2.2. Calculation of the Mutual Inductance Effective Permeability

The calculation of the second step of the above explained method is solving the Maxwell's equations numerically. The system of equations can be simplified, so that the time varying parts of the equations can be neglected as well as the electric field calculation. This simplifies the problem to static magnetic problem, which is represented in Figure 1. with the following governing equations:

$$\nabla \times H = J , \tag{3}$$

$$\nabla \cdot B = 0 , \tag{4}$$

in region Ω_0 and Ω_m, where H is the magnetic field intensity, B is the magnetic flux density, J is the source current density of the exciting coil, which is equal to zero during the calculation of the mutual inductance effective permeability.

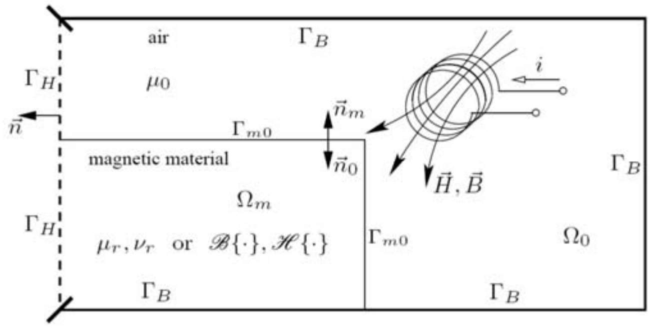

Figure 1. Scheme of static magnetic field problems.

The constitutive relations between magnetic field intensity and magnetic flux density are as follows:

$$B = \mu_0 H \text{ in region } \Omega_0, \tag{5}$$

$$B = \mu_0 \mu_r H \text{ in region } \Omega_m, \tag{6}$$

where μ_0 is the permeability of vacuum, and μ_r is the relative permeability of magnetic material, assuming linear relation between magnetic field intensity and magnetic flux density in all problem region [6].

The problem region is surrounded by the boundary Γ_H and Γ_B. On Γ_H the tangential component of H, on Γ_B the normal component of B is set to zero, respectively, i.e. $H \times n = 0$ on Γ_H, and $B \cdot n = 0$ on Γ_B [6].

The magnetic vector potential A can be introduced as follows:

$$B = \nabla \times A . \tag{7}$$

Substituting equation (7) into (3) and using the constitutive relation the following equation can be obtained

$$\nabla \times (\nu \nabla \times A) = J , \tag{8}$$

where ν is the magnetic reluctivity.

During the calculation of the mutual inductance effective permeability the source current density is zero, because the known external magnetic field strength H_E can be defined as a Neumann boundary condition. Due to this the partial differential equation that shall be solved is the following:

$$\nabla \times (\nu \nabla \times A) = 0 , \tag{9}$$

which is the well known Laplace's equation [7].

The computation of (9) is easy and can be performed by several numerical methods.

It is important to note that the prescribed external magnetic field (H_E) on the boundary represents the known magnetic field during the sensitivity measurement, which is created by the Helmholtz-coil. Solution of the above described problem can be seen in Figure 2.

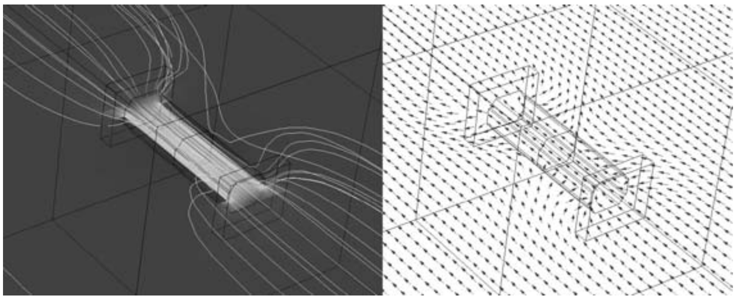

Figure 2. B and H field of the ferrite core represented by streamlines and arrow plot

3. Calculation of Sensitivity

3.1. Sensitivity calculation using the mutual inductance effective permeability

After the mutual inductance effective permeability is computed the sensitivity can be calculated as follows.

The excitation current of the Helmholtz-coil and so the magnetic flux density inside the coil during the sensitivity measurement is known and can be written as follows:

$$I_E(t) = \hat{I}_E \sin(\omega t) , \tag{10}$$

$$B_E(t) = \hat{B}_E \sin(\omega t) , \tag{11}$$

where I_E and B_E, are the source current and the magnetic flux density of the Helmholtz coil. The $\hat{B}_E$ magnetic flux density peak value can be determined by [8]

$$\hat{B}_E = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N_H \hat{I}_E}{R} , \tag{12}$$

where N_H and R, are the number of turns and diameter of the Helmholtz-coil.

The induced voltage in coils in general can be calculated by the Farraday's law of induction:

$$U_i = -N \frac{d\Phi}{dt}, \tag{13}$$

where *Φ* is the total flux which flows through the inner cross section area of the coil.

Substituting (11) and (13) in (1) the following equation can be obtained:

$$S(t) = \frac{-N\hat{B}_m \cos(\omega t)\omega A}{\hat{B}_E \sin(\omega t)}, \tag{14}$$

where A and B_m is the inner cross section area, and the magnetic flux density of the Transponder coil.

As only the peak value of the sensitivity is interesting from the calculation point of view, the time varying components can be eliminated, as follows:

$$S_{\max} = \frac{N\hat{B}_m \omega A}{\hat{B}_E}. \tag{15}$$

The B_m magnetic flux density inside the ferrite core of the Transponder coil can be calculated using (2). Using this substitution the (15) can be represented as follows:

$$S_{\max} = \frac{N\hat{B}_E \mu_{M\_eff} \omega A}{\hat{B}_E} = N\mu_{M\_eff}\omega A. \tag{16}$$

According to the equation (16) the sensitivity value as defined in (1) is directly proportional to the number of turns, the inner cross section area, the mutual inductance effective permeability of the transponder coil, and the angular frequency of the system.

Once the mutual inductance effective permeability is known – which is defined only by the geometry and the used materials – the sensitivity can be calculated for any inductance value at any frequency.

3.2. Specific sensitivity referred to self inductance

The mutual inductance related parameters of the inductor construction has the highest influence on the quality of the coupling in the inductively coupled systems, but beside those the self inductance related parameters are also important. This is the reason why the mutual inductance effective permeability and the sensitivity shall have a specific value referred to the self inductance value. The specific sensitivity referred to the self inductance can be calculated as follows:

$$S_L = \frac{\mu_{M\_eff} A}{\sqrt{Al}}, \tag{17}$$

where S_L is the specific sensitivity referred to self inductance, A is the cross section area of the magnetic core, and Al is the self inductance factor (Al represents the self inductance value of an inductor with one wire turn with the specific magnetic core).

The specific sensitivity is used for comparing different inductor constructions. It makes completely different inductor constructions comparable from the coupling point of view. The higher the specific sensitivity value, the higher the sensitivity and the better the coupling of the inductor is.

The sensitivity for a particular inductance value with a given inductor construction (magnetic core) can be calculated using the specific sensitivity value as follows:

$$S = S_L \omega \sqrt{L} \,, \tag{18}$$

where ω is the angular frequency and L is the self inductance.

4. Verification of the calculation method

The sensitivity calculation by the mutual inductance effective permeability method was verified with market available RFID transponder coils in two different SMT (Surface Mount Technology) sizes. In both size (11mm and 8mm) not only one inductance but the full inductance range were tested [9].

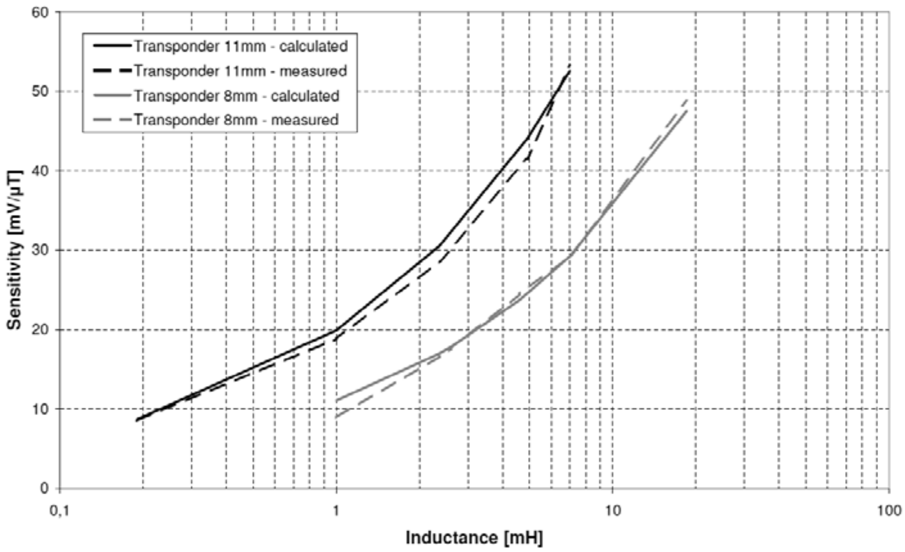

Figure 3. Comparison of calculated and measured sensitivity values for 11mm and 8mm Transponder coils in inductance range.

After the mutual inductance effective permeability and the specific sensitivity referred to inductance values were calculated for both inductor constructions, the sensitivity for different inductance values on different frequencies were calculated. The calculated sensitivity values were in good agreement with the measured sensitivity values in the full inductance range, Figure 3., as well as in frequency range, Figure 4.

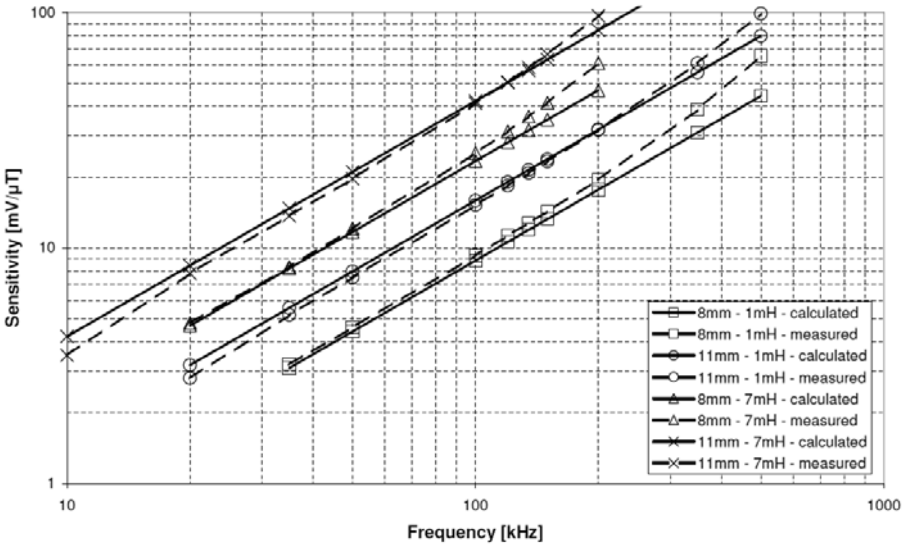

Figure 4. Comparison of calculated and measured sensitivity values for 11mm and 8mm Transponder coils in frequency range.

5. Conclusion

The introduced mutual inductance effective permeability parameter gives information about the performance of the inductor design in inductively coupled applications. A calculation method was presented that use the mutual inductance effective permeability for the sensitivity calculation. The calculation of the mutual inductance effective permeability due to the simplicity of the static magnetic problem type is cheap in terms of time and hardware requirements.

The presented calculation method is an effective and accurate technique which was verified in market available transponder coils. The calculated and measured sensitivity values were in good agreement which confirms the utility of the mutual inductance effective permeability method.

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