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Research Article

Impact of a magnetic field on the onset of Jeffrey nanofluid

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Abstract: In this paper, we address the problem of magneto-convection in a Jeffery nanofluid layer confined between infinite horizontal free-free boundaries, heated from below. The Navier-Stokes equations are modified to combine the Jeffrey parameter, nanoparticles and magnetic field term. A linear stability analysis is carried out using the perturbation technique, normal mode analysis and the Galerkin-type weighted residual method, leading to the derivation of a dispersion relation for the Rayleigh number. Numerical and graphical results explained the effects of key parameters on thermal convection of non-Newtonian nanofluid, including parameters—the modified diffusivity ratio, Lewis number, nanoparticles Rayleigh number, Jeffrey parameter and Chandrasekhar number, on the onset of stationary convection. The study reveals that the magnetic field is having a stabilizing effect, thereby delaying the onset of stationary convection in the nanofluid layer.

Keywords: *Convection; Nanofluid; Magnetic field; Jeffrey Model*

Nomenclature

q	heat flux $\left(Jm^{-2}s^{-1}\right)$
dT/dx	temperature gradient $\left(Km^{-1}\right)$
d	distance between the two horizontal boundaries (m)
$H = H(0,0,H)$	vertical magnetic field $\left(Am^{-1}\right)$
$\vec{g} = -g\hat{e}_z$	acceleration due to gravity $\left(ms^{-1}\right)$
$\hat{e}_z$	unit vector in z-direction
T	temperature (K)
K	kelvin
T_0	temperature at the lower boundary (K)
T_1	temperature at the upper boundary
P	pressure (Pa)
$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k$	gradient (m^{-1})
$D_B = \frac{K_B T}{3\pi\mu d_p}$	Brownian diffusion coefficient $\left(m^2 s^{-1}\right)$
(h_x, h_y, h_z)	magnetic field $\left(Am^{-1}\right)$
$(D/Dt) = (\partial/\partial t) + \vec{V}.\nabla$	material derivative
$\vec{V}$	nanofluid's velocity (ms^{-1})
k	medium permeability $\left(Hm^{-1}\right)$

ρ_P	nanoparticle mass density $\left(Kgm^{-3}\right)$
K_B	Boltzmann's constant $(J\kappa^{-1})$
k_x	wave number in x-direction $\left(m^{-1}\right)$
k_y	wave number in y-direction $\left(m^{-1}\right)$
n	growth rate $\left(s^{-1}\right)$
a	wave number $\left(m^{-1}\right)$
D_T	Thermophoretic coefficient $\left(ms^{-1}\right)$
$R_m = \frac{\left(\rho_P\phi_0 + \rho_0\left(1-\phi_0\right)\right)gd^3}{\mu\alpha}$	Basic density Rayleigh number
$R_a = \frac{\rho g\beta d^4\left(T_0 - T_1\right)}{\mu\alpha}$	thermal Rayleigh number
$R_n = \frac{\left(\rho_P - \rho_0\right)\left(\phi_1 - \phi_0\right)}{\mu\alpha} gd^3$	Nanoparticle number
Q	Chandrasekhar number
$\mathrm{Pr}_1 = \mu_0 / \left(\rho_0\alpha\right)$	Prandtl number
$\mathrm{Pr}_1 = \mu_0 / \left(\rho_0\eta\right)$	magnetic Prandtl number
$N_A = \frac{D_T\left(T_0 - T_1\right)}{D_B T_1\left(\phi_1 - \phi_0\right)}$	modified diffusivity ratio
$N_B = \frac{\left(\rho c\right)_P}{\left(\rho c\right)_f}\left(\phi_1 - \phi_0\right)$	modified particle density increment
L_e	Lewis number
Subscripts	
b	basic state solution
Superscript	
*	non dimensional quantities
'	perturbation quantities
Greek symbols	
ϕ	nanoparticle's volume fraction
ϕ_0	volumetric fraction at the lower boundary
ϕ_1	volumetric fraction at the upper boundaries
ρ_0	density of the fluid at reference temperature
μ	viscosity of the fluid $\left(Kgm^{-1}s^{-1}\right)$

λ	Jeffrey parameter
$(\rho c)_P$	specific heat at constant pressure
μ_e	magnetic conductivity $\left(Kgms^{-2}A^{-2}\right)$
η	magnetic diffusivity $\left(m^2s^{-1}\right)$
α	thermal diffusivity $\left(m^2s^{-1}\right)$
ρc	fluid's heat capacity $\left(JKg^{-1}K^{-1}\right)$
$(\rho c)_f$	thermal heat capacity of nanoparticles $\left(JKg^{-1}K^{-1}\right)$
κ	thermal conductivity $\left(WmK^{-1}\right)$
ζ	vorticity
ξ	current density
β	coefficient of thermal expansion

I. INTRODUCTION

Fluids are widely used as heat carriers in heat transfer equipment primarily because they can transfer heat effectively through convection. Although gases also exhibit convection, their low density makes heat transport less efficient compared to liquids. Heat transfer forms the foundation of modern technology due to its extensive applications in areas such as the cooling of electronic devices, power generation, chemical and industrial processes, geothermal systems, biomass reactors, aerospace engineering, etc. It also plays a vital role in everyday appliances such as refrigerators, air conditioners, heaters, ovens, irons, etc. The thermal conductivity of a material is a key property in designing heat transfer equipment, as it determines how easily heat can flow through the substance. According to Fourier's law of heat conduction:

$$q = -\kappa(\frac{dT}{dx})$$

where, q is the heat flux (rate of heat flow per unit area), κ is thermal conductivity and dT/dx is the temperature gradient. This implies that heat flux is directly proportional to the thermal conductivity and the temperature gradient. In general, the thermal conductivity of solids is greater than that of fluids at room temperature. However, fluids are still preferred as heat carriers in many heat transfer systems because they can easily conform to the shape of a container and transfer heat via both conduction and convection. The concept of nanofluids was first introduced by Choi [1], who proposed a new class of heat transfer fluids created by suspending metallic nanoparticles (such as Au, Ag, Cu or oxide ceramics $\left(Al_2O_3, CuO\right)$, Carbide ceramics, etc.) in conventional bases like water, ethylene glycol, engine oil, etc. We expected these nanofluids to exhibit higher thermal conductivity compared to traditional heat transfer fluids. Subsequent experimental studies by Masuda et al. [2] and Eastman et al. [3] confirmed that the thermal conductivity of nanofluids increases with a rise in nanoparticles volume fraction and a decrease in particle size. Modern advanced nanotechnology now enables the production of nanoparticles with sizes around 100 nm. The study of thermal instability in nanofluids has become increasingly important because of their wide-ranging applications in areas such as nuclear reactor cooling, heat exchanger systems, cancer treatment, heating down of electronic gadgets, enhancing thermal management in vehicles, etc. Bénard [4] was the first to experimentally examine the thermal stability of a flat fluid layer heated from the bottom and later Lord Rayleigh [5] provided the theoretical foundation for this phenomenon in regular fluids. Magnetoconvection refers to convection in an electrically conducting fluid that is kept between two horizontal planes under a uniform magnetic field, which may be oriented vertically or horizontally. The effect of a magnetic field on Rayleigh–Bénard convection in nanofluids is of great interest due to its applications in chemical and biochemical engineering, various industrial processes and several geophysical and astrophysical phenomena.

In the classical Rayleigh–Bénard setup, magneto-convection in a nanofluid results from the combined effects of thermal buoyancy and the Lorentz force produced by the applied magnetic field. The Lorentz force introduces an additional dimensionless

parameter, the Chandrasekhar number**,** which measures the influence of the magnetic field on the onset and behaviour of convection. Chandrasekhar [6] studied the combined effects of magnetic field and rotation on convection in regular nanofluids, while Vadasz [7] extended this analysis to porous media. They used the classical mathematical technique, named normal mode analysis. It helps to determine whether the small disturbance grow, decay or remain same with the time. Nanofluids exhibit a variety of exhibit a variety of behaviours and among them the Jeffrey nanofluid—a non-Newtonian, viscoelastic fluid—is widely used in biomedical flows, industrial applications, etc. G.B. Jeffery [8] was a fluid mechanician who introduced a viscoelastic model to describe the complex flow behaviour in fluids that do not obey Newton's law of viscosity. Jeffrey fluid introduced the Jeffrey parameter which is the ratio of the relaxation time to the retardation time. Considering the effects of thermophoresis and Brownian diffusion, Buongiorno [9] proposed a model for convective transport in nanofluids. Boussinesq [10] remarked that the variations of density and other parameters like viscosity, specific heat, etc., can be ignored except where they are multiplied by the accelerating gravity in the equation of motion. Zeytounian [11] gave a contemporary view of Boussinesq approximation. Tzou [12, 13] investigated natural convection instability in nanofluids and found that their critical Rayleigh number could differ from that of regular fluids by one or two orders of magnitude. Nield and Kuznetsov [14] analytically examined the onset of convection in a finite layer of nanofluid and concluded that for nanofluid with a large Lewis number, the main contribution of the nanoparticles arises through buoyancy coupled with nanoparticle concentration conservation. Based on this framework, several researchers [15-17] analysed the thermal instability of nanofluids in porous media using linear stability theory. Gupta et al. [18] explored the effect of a magnetic field on a nanofluid layer without a porous medium and showed that the Chandrasekhar number has a stabilizing influence on convection. Yadav et al. [19] studied the combined effects of rotation and magnetic field on the stability of a nanofluid layer under different boundary conditions—free–free**,** rigid–free and rigid–rigid. The results indicated that the critical Rayleigh number for the onset of convection in nanofluid is lower than that for regular fluids under the same magnetic and rotational parameters. Sharma et al. [20] examined magneto-thermosolutal convection in a Jeffrey nanofluid saturated within a porous medium using the Darcy model represent and the normal-mode analysis**.** Their findings indicated that the magnetic field stabilizes the system, whereas the Jeffrey parameter—defined as the ratio of relaxation time to retardation time—tends to destabilize it. Rani and Tomar [21] analysed the thermal conductivity of micropolar fluids considering the influence of the Hall current and reported that the Hall current produces a destabilizing effect on the system. Yadav et al. [22] explored the thermal convection behaviour of Kuvshiniski viscoelastic nanofluid in a porous medium under more physically realistic boundary conditions. Devi et al. [23] studied the impact of an internal heat source on Darcy–Brinkman convection within a Casson nanofluid layer and found that both the non-Newtonian parameters and intensity of the internal heat source significantly enhance system instability. Chand et al. [24] investigated the thermal instability of a rotating couple– stress nanofluid using the Darcy model to represent the porous medium and observed that oscillatory convection does not occur under their assumptions. Sharma et al. [25] investigated entropy generation, as well as heat and mass transfer, in Jeffrey fluid flow under the influence of solar radiation. The study examined the combined effects of motile gyrotactic microorganisms and copper nanoparticles in a polyvinyl alcohol–water base fluid. It further considered key source terms—viscous dissipation, Ohmic heating, exponential heat source and chemical reaction—on electro magneto hydrodynamic (EMHD) Jeffrey fluid flow. Sharma et al. [26] examined thermosolutal convection in a Jeffrey nanofluid saturated with a porous medium under the effect of a magnetic field. Using the Darcy model and the normal mode technique, they concluded that the magnetic field exerts a stabilizing influence, whereas the Jeffrey parameter, defined as the ratio of relaxation time to retardation time, tends to destabilize the nanofluid layer during stationary convection. Bains et al. [27] considered convection in a Casson nanofluid containing suspended particles and employed the Darcy-Brinkman model to account for the porous medium. Sharma et al. [28] further investigated the combined effects of magnetic field and rotation on a Jeffrey nanofluid layer using the Brinkman model for non-oscillatory convection and found that both factors contribute to system stabilization. In the present work, nanoparticles are assumed to be dispersed within the nanofluid using surfactant or surface charge technology. The study focuses on the thermal instability of a nanofluid layer subjected to a magnetic field with free boundaries, assuming that the nanoparticles accumulate near the bottom of the layer. The critical Rayleigh number and the corresponding wavenumber are determined using Galerkin–type weighted residual method. The Galerkin method, introduced by Boris Galerkin in 1915, is a mathematical approach for obtaining approximate solutions of differential equations, especially those arising in fluid mechanics, heat transfer and stability analyses. In stability problems, this approach transforms the differential eigenvalue equations into a system of algebraic equations through appropriate trial functions, allowing the determination of the critical Rayleigh number or the

stability parameters. Although extensive research has been carried out on convection phenomena in nanofluids within porous media, studies focusing on nanofluid magnetoconvection without porous structures remain limited. To address this research gap, the present study investigates magnetoconvection in a nanofluid layer without a porous medium to cover the gap. As per our literature review, the problem we have taken has not been discussed yet.

II. Physical Model Of The Formulated Problem

We consider an endless flat layer of the Jeffrey nanofluid placed between two planes with a certain vertical thickness $'d'$ from each other, exposed to a steady magnetic field $H = H(0,0,H)$ in the absence of porous media and subjected to heating from below under the effect of gravity $\vec{g} = -g\hat{e}_z$ acting vertically downwards along the z-direction (as displayed in **Fig. 1**). The nanoparticle volume fraction and temperature stay the same at the edges, meaning at the top and bottom, with specific values, i.e., at the lower boundary $T = T_0$; $\phi = \phi_0$ and at the upper boundary $T = T_1, \phi = \phi_1$ with the condition $T_0 > T_1$ and $\phi_0 > \phi_1$. Where T_0, ϕ_0 represent the temperature and volumetric fraction at the lower boundary, respectively, and T_1, ϕ_1 represent the temperature and volumetric fraction at the upper boundary, respectively. As the number nanoparticles is more at the bottom, so we are having bottom-heavy setup and we are considering the situation with free-free boundaries.

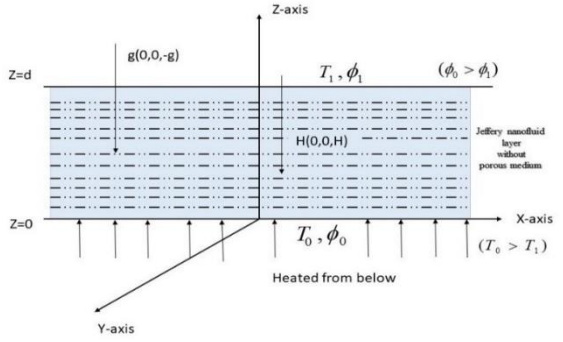


Figure 1. *Physical model of the formulated Problem.*

III. Governing Equations

The governing equations are taken from the work of Buongiorno [9], Tzou [12, 13], Chandrasekhar [6] and Gupta et al. [18] after applying the Boussinesq approximation [10] as:

$$\nabla.\vec{V} = 0, \tag{1}$$

$$\rho_0 \frac{D\vec{V}}{Dt} + \nabla P - \left(\frac{\mu}{1+\lambda}\right)\nabla^2\vec{V} - \rho\vec{g} - \frac{\mu}{4\pi}(\nabla \times h) \times \mathrm{H} = 0, \tag{2}$$

$$(\rho c)\frac{DT}{Dt} - \mathrm{K}\nabla^2 T - \left(\frac{D_T}{T_1}\right)\nabla T.\nabla T - D_B(\rho c)_P(\nabla\phi.\nabla T) = 0, \tag{3}$$

$$\frac{D\phi}{Dt} - D_B\nabla^2\phi - \left(\frac{D_T}{T_1}\right)\nabla^2 T = 0. \tag{4}$$

In the above–described equations, the quantity $V = u\hat{e}_x + v\hat{e}_y + w\hat{e}_z$ is the nanofluid's velocity, $\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{V}.\nabla$ is the material derivative, the symbol ϕ denotes the volumetric fraction of the nanoparticles, the quantity D_B indicates the diffusion coefficient, D_T corresponds to the thermal diffusion coefficient, μ is fluid's viscosity, $h = h_x\hat{e}_x + h_y\hat{e}_y + h_z\hat{e}_z$ represents the component of magnetic field, the fluid's heat capacity is represented by (ρc), $(\rho c)_f$ represents the thermal capacity of the nanoparticles, μ_e indicates the magnetic conductivity and κ represents the conductivity constant of the system.

Maxwell equations are given by;

$$\left(\frac{\partial}{\partial t} + \vec{V}.\nabla\right)h = (H.\nabla)\vec{V} + \eta\nabla^2 h. \tag{5}$$

$$div(h) = 0, \tag{6}$$

and η is the magnetic diffusivity.

The Boundary conditions according to this problem are given by;

$$\left.\begin{aligned} W = 0,\ T = T_0,\ \phi = \phi_0 \ \ when\ z = 0, \\ W = 0,\ T = T_1,\ \phi = \phi_1 \ \ when\ z = d. \end{aligned}\right\} \tag{7}$$

IV. Non-Dimensional Analysis

Now, we proceed by introducing non-dimensional quantities given below:

$$\left.\begin{aligned} (x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right),\ P^* = \frac{d^2 P}{\alpha\mu}, \\ (u^*, v^*, w^*) = \left(\frac{ud}{\alpha}, \frac{vd}{\alpha}, \frac{wd}{\alpha}\right), t^* = \frac{t\alpha}{d^2}, \\ \nu = \frac{\mu}{\rho_0}, h^* = \frac{\eta}{\mathrm{H}\alpha}h,\ \phi^* = \left(\frac{\phi - \phi_1}{\phi_0 - \phi_1}\right), \\ T^* = \left(\frac{T - T_1}{T_0 - T_1}\right),\ \nabla^* = d\nabla. \end{aligned}\right\} \tag{8}$$

where '*' represents the non-dimensional quantities.
Now, substituting equation (8) into the equations (1)– (4) and ignoring the symbol '*' for simplicity, we get the non-dimensional representation of the equations as:

$$\nabla.\vec{V} = 0, \tag{9}$$

$$\frac{1}{\mathrm{Pr}_1}\frac{D\vec{V}}{Dt} + \nabla P - \frac{1}{1+\lambda}\nabla^2\vec{V} + R_m\hat{e}_z - R_a T\hat{e}_z + R_n\phi\hat{e}_z - Q\left\{\left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x}\right)\hat{e}_x + \left(\frac{\partial h_y}{\partial z} - \frac{\partial h_z}{\partial y}\right)\hat{e}_y\right\} = 0, \tag{10}$$

$$\frac{DT}{Dt} - \nabla^2 T - \left(\frac{N_A N_B}{L_e}\right)\nabla T.\nabla T - \left(\frac{N_B}{L_e}\right)\nabla\phi.\nabla T = 0, \tag{11}$$

$$\frac{D\phi}{Dt} - \left(\frac{1}{L_e}\right)\nabla^2\phi - \left(\frac{N_A}{L_e}\right)\nabla^2 T = 0, \tag{12}$$

$$div(h) = 0, \tag{13}$$

$$\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}.\frac{dh}{dt} = \left(\frac{\partial u}{\partial z}\hat{e}_x + \frac{\partial v}{\partial z}\hat{e}_y + \frac{\partial w}{\partial z}\hat{e}_z\right) + \nabla^2 h, \tag{14}$$

where the dimension-free parameters are:

$\mathrm{Pr}_1 = \frac{\nu}{\alpha}$ (Prandtl number);

$\mathrm{Pr}_2 = \frac{\nu}{\eta}$ (the Magnetic Prandtl number);

$L_e = \frac{\alpha}{D_B}$ (Lewis number);

$R_a = \frac{\rho g \beta d^4 (T_0 - T_1)}{\mu\alpha}$ (Thermal Rayleigh number);

$Q = \frac{\mu_e H^2 d^2}{4\pi\eta\mu}$ (Chandrasekhar number);

$R_m = \frac{g d^3 (\rho_P\phi_0 + \rho_0(1-\phi_0))}{\mu\alpha}$ (Basic Density Rayleigh number);

$R_n = \frac{(\phi_1 - \phi_0)(\rho_P - \rho_0) g d^3}{\mu\alpha}$ (Nanoparticles Rayleigh number);

$N_A = \frac{D_T (T_0 - T_1)}{T_1 D_B (\phi_1 - \phi_0)}$ (Modified Diffusivity ratio);

$N_B = \left(\frac{(\rho c)_p}{(\rho c)}\right)(\phi_1 - \phi_0)$ (Modified Particle Density increment).

Where the symbols, $\hat{e}_x$, $\hat{e}_y$ and $\hat{e}_z$ are the unit vectors along the x-, y- and z-direction, respectively.

The dimension–free boundary conditions are given as:

$$\left.\begin{aligned}(\phi, w, T) &= (1,0,1) \ when \ z = 0\\ (\phi, w, T) &= (0,0,0) \ when \ z = 1\end{aligned}\right\}. \tag{15}$$

V. Basic State Solutions

At the initial state the fluid is at rest, i.e., its initial velocity is zero and variables defining the initial state are given as:

$$(u, v, w, h) = (0,0,0,0),$$
$$(\phi, T, P) = \left[\phi_b(z), T_b(z), P_b(z)\right]. \tag{16}$$

where the subscript "b" is representing the value of the variable at the basic state.

Applying equation (16) into equations (10) -(12), we get:

$$0 = -\frac{dP_b(z)}{dz} - R_m + R_a T_b(z) - R_n\phi_b(z), \tag{17}$$

$$0 = \frac{d^2 T_b(z)}{dz} + \frac{N_B}{L_e}\frac{dT_b(z)}{dz}\left(\frac{d\phi_b(z)}{dz} + N_A\frac{dT_b(z)}{dz}\right), \tag{18}$$

$$0 = \frac{d^2\phi_b}{dz} + N_A\frac{d^2 T_b}{dz}, \tag{19}$$

On solving equation (19) and using the boundary conditions (15), we get

$$\phi_b = (1+N_A)(1-z) - N_A T_b, \tag{20}$$

Now substituting equation (20) in equation (18), we get

$$0 = \frac{d^2 T_b(z)}{dz} + \frac{N_B}{L_e}(1+N_A)\left(\frac{dT_b(z)}{dz}\right)^2,$$

Solving the above equation and using the boundary condition (15), we get

$$T_b(z) = \frac{1 - e^{-(1-z)L}}{1 - e^{-L}}, \tag{21}$$

where $L = \frac{(1+N_A)N_B}{L_e}$.

According to Buongiorno [8], for most of the nanofluids investigated, the parameter N_A is typically of the order 10, while the ratio $L_e/(\phi_0 - \phi_1)$ is of the order $10^5 - 10^6$. In addition, the nanoparticle volume fraction decrement is of the order $10^2 - 10^3$. From this information, it can be

concluded that L_e is relatively large, indicating that more heat is required to initiate convection. This behaviour is attributed to the high thermal conductivity of the nanofluids. Therefore, the exponential term in equation (21) is small and by expanding it into a power series and retaining only the first-order terms, the approximated solutions can be obtained as:

$$T_b = 1 - z,\ \phi_b = 1 - z. \tag{22}$$

Which resembles the solution obtained by Gupta et al. [17].

VI. Perturbation Equations

Now, to investigate the stability of the considered problem, we add very small disturbances as given below:

$$\begin{aligned} &V(u,v,w) = (0,0,0) + V'(u',v',w'),\\ &P = P_b + P',\ T = T_b + T',\ \phi = \phi_b,\\ &h(h_x,h_y,h_z) = \mathrm{H}(0,0,\mathrm{H}) + h'(h'_x,h'_y,h'_z), \end{aligned} \tag{23}$$

Substituting equation (23) into the equations (9)–(13) and by ignoring the asterisks $'$ for our convenience, we get

$$\nabla.\vec{V} = 0\,, \tag{24}$$

$$\begin{aligned} &\frac{1}{\mathrm{Pr}_1}\left(\frac{\partial \vec{V}}{\partial t}\right) = -\nabla P + \frac{1}{1+\lambda}\nabla^2\vec{V} + R_a T\hat{e}_z - R_n\phi\hat{e}_z\\ &+Q\left[\left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x}\right)\hat{e}_x - \left(\frac{\partial h_z}{\partial x} - \frac{\partial h_y}{\partial z}\right)\hat{e}_y\right], \end{aligned} \tag{25}$$

$$\frac{\partial T}{\partial t} - W = \nabla^2 T - \frac{N_B}{L_e}\left(\frac{\partial T}{\partial z} + \frac{\partial \phi}{\partial z}\right) - 2\frac{N_A N_B}{L_e}\frac{\partial T}{\partial z}, \tag{26}$$

$$\frac{\partial \phi}{\partial t} - W = \frac{1}{L_e}\nabla^2\phi + \frac{N_A}{L_e}\nabla^2 T_b, \tag{27}$$

$$div(h) = 0, \tag{28}$$

where W is the z–component of velocity.

Now equating the x, y and z–components of equation (25) on both sides, we get

$$\frac{1}{\mathrm{Pr}_1}\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{1}{1+\lambda}\nabla^2 u + Q\left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x}\right), \tag{29}$$

$$\frac{1}{\mathrm{Pr}_1}\frac{\partial v}{\partial t} = -\frac{\partial P}{\partial y} + \frac{1}{1+\lambda}\nabla^2 v - Q\left(\frac{\partial h_z}{\partial y} - \frac{\partial h_y}{\partial z}\right), \tag{30}$$

$$\frac{1}{\mathrm{Pr}_1}\frac{\partial w}{\partial t} = -\frac{\partial P}{\partial z} + \frac{1}{1+\lambda}\nabla^2 w + R_a T - R_n\phi\,, \tag{31}$$

Now eliminating the pressure term in equations (29) and (30), we get

$$\begin{aligned} &\frac{1}{\mathrm{Pr}_1}\frac{\partial}{\partial t}\left(\nabla^2 W\right) - \frac{1}{1+\lambda}\nabla^4 - R_a\nabla^2{}_H\\ &+R_n\nabla^2{}_H\phi - Q\nabla^2\frac{\partial h_z}{\partial z} = 0. \end{aligned} \tag{32}$$

Now multiplying equation (29) by $-\frac{\partial}{\partial y}$ and equation (30) by $\frac{\partial}{\partial x}$ and adding the resultant equations, we get

$$\frac{1}{\mathrm{Pr}_1}\frac{\partial}{\partial t}(\zeta) = \frac{1}{1+\lambda}\nabla^2\zeta + Q\left(\frac{\partial \xi}{\partial z}\right), \tag{33}$$

Now equating the x-, y- and z-component of equation (14) on both sides, we get

$$\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\frac{\partial h_x}{\partial t} = \frac{\partial u}{\partial z} + \nabla^2 h_x\,, \tag{34}$$

$$\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\frac{\partial h_y}{\partial t} = \frac{\partial v}{\partial z} + \nabla^2 h_y\,, \tag{35}$$

$$\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\frac{\partial h_z}{\partial t} = \frac{\partial w}{\partial z} + \nabla^2 h_z\,. \tag{36}$$

Now multiply equation (34) by $\frac{\partial}{\partial x}$ and (35) by $\frac{\partial}{\partial y}$, after that, by adding them, we get

$$\begin{aligned} &\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\frac{\partial}{\partial t}\left(\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}\right) =\\ &\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \nabla^2\left(\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}\right) \end{aligned}, \tag{37}$$

$$\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\frac{\partial}{\partial t}(\xi) = \frac{\partial}{\partial z}(\zeta) + \nabla^2\xi\,. \tag{38}$$

where ξ is the current density and ζ is the vorticity.

VII. Normal Mode Analysis

In terms of normal modes, the disturbed quantities can be written as:

$$\begin{aligned} &(W,T,\phi,h_z,\xi,\zeta) =\\ &\left[W(z),\theta(z),\phi(z),K(z),X(z),Z(z)\right]\\ &\mathrm{e}^{(ik_x x + ik_y y + nt)}, \end{aligned} \tag{39}$$

where the quantities k_x and k_y indicate the wave number along the x-direction and y-direction, respectively and n represents the growth rate of the perturbation.

Now making use of the equation (39) in the equations (32), (33), (37), (38), (26) and (27) we get the following equations:

$$\left(D^2-a^2\right)\left(\frac{1}{1+\lambda}\left(D^2-a^2\right)-\frac{n}{\mathrm{Pr}}\right)W + a^2\left(R_n\Phi-R_a\Theta\right)+Q\left(D^2-a^2\right)DK=0. \tag{40}$$

$$\frac{1}{1+\lambda}\left(D^2-a^2\right)Z+QDX=\frac{n}{\mathrm{Pr}_1}Z\,, \tag{41}$$

$$DZ+\left(D^2-a^2\right)X=\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}nX\,, \tag{42}$$

$$DW+\left(D^2-a^2\right)K=\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}nK\,, \tag{43}$$

$$n\theta-W-\left(D^2-\frac{N_B}{L_e}D-2\frac{N_AN_B}{L_e}D-a^2\right)\Theta + \left(\frac{N_B}{L_e}\right)D\Phi=0. \tag{44}$$

$$n\Phi-W-\frac{N_A}{L_e}\left(D^2-a^2\right)\Theta - \left(\frac{1}{L_e}\right)\left(D^2-a^2\right)\Phi=0 \tag{45}$$

The conditions for the boundaries of the nanofluid layer are given as;

$$W=\Theta=\Phi=0 \;\; at \;\; z=0 \; and \; z=1. \tag{46}$$

Eliminating K, X *and* Z from the equations (40)–(43), we get

$$\begin{pmatrix}\left(D^2-a^2\right)\left(\frac{\left(D^2-a^2\right)}{1+\lambda}-\frac{n}{\mathrm{Pr}}\right)\left(\left(D^2-a^2\right)-\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}n\right)\\ -QD^2\left(D^2-a^2\right)\end{pmatrix}W + \left(D^2-a^2-n\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\right)\left(R_na^2\Phi-R_aa^2\Theta\right)=0. \tag{47}$$

Equations (47), (44) and (45), together with the boundary conditions (46), forms the double eigenvalue problem.

By the Galerkin weighted residual method, the approximate solutions are given by:

$$(W,\Theta,\Phi)=(W_0,\,\Theta_0,\Phi_0)\,\mathrm{Sin}\,\pi z\,. \tag{48}$$

By substituting equation (48) into the equations (47), (44) and (45) and integrating from 0 to 1 with the use of orthogonality condition, we get the following system of homogeneous equations in the matrix form as:

$$\begin{bmatrix}A & B & C\\ 1 & D & 0\\ 1 & E & F\end{bmatrix}\begin{bmatrix}W_0\\ \Theta_0\\ \Phi_0\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}. \tag{49}$$

where,

$$A=\left(\frac{J^2}{1+\lambda}+\frac{n}{\mathrm{Pr}_1}J\right)\left(J+n\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\right)-JQ\pi^2\,,$$

$$B=-R_aa^2\left(J+n\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\right),$$

$$C=R_na^2\left(J+n\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\right),$$

$$D=-\left(J+n\right),$$

$$E=-\left(\frac{N_A}{L_e}J\right),$$

$$F=-\left(\frac{1}{L_e}J+n\right),$$

The above equation has a nonzero solution if the coefficient matrix in equation (49) has full rank. Further, by solving it, we get the following expression as:

$$R_a=\frac{\left\{\left(\frac{J^2}{1+\lambda}+\frac{nJ}{\mathrm{Pr}_1}\right)\left(J+n\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\right)+QJ\pi^2\right\}\left\{J+n\right\}}{a^2\left(J+n\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}\right)} + \frac{\left\{R_n\left[J+n-\left(\frac{N_A}{L_e}\right)J\right]\right\}}{\frac{1}{L_e}J+n}, \tag{50}$$

where $J=\pi^2+a^2$.

To find the thermal Rayleigh number at the marginal state, we put $n=0$ in the equation (50) and we get:

$$\left(R_a\right)_S=\frac{\left(\pi^2+a^2\right)}{a^2}\left(\frac{\left(\pi^2+a^2\right)^2}{1+\lambda}+Q\pi^2\right). + R_n\left[L_e-N_A\right], \tag{51}$$

Equation (51) gives the value of the Rayleigh number for the considered stationary mode.

Letting $a^2=\pi^2x$. Now equation (49) becomes

$$\left(R_a\right)_S=\frac{\pi^4\left(1+x\right)}{x}\left(\frac{\left(1+x\right)^2}{1+\lambda}+\frac{Q}{\pi^2}\right) + R_n\left[L_e-N_A\right], \tag{52}$$

Now, differentiating the above equation w.r.t. x and equating it equal to 0, we get

$$2x^3+3x^2-\left(\frac{Q\left(1+\lambda\right)}{\pi^2}+1\right)=0\,. \tag{53}$$

Now the real root of equation (53) has been calculated using Mathematica software and is given by

$$x=-\frac{1}{2}+\frac{3}{2^{\frac{2}{3}}\left(G+\sqrt{-2916+G}\right)^{\frac{1}{3}}}+\frac{\left(G+\sqrt{-2916+G}\right)^{\frac{1}{3}}}{6(2)^{\frac{1}{3}}}, \tag{54}$$

where,

$$G=54+\frac{108Q}{\pi^2}+\frac{108LQ}{\pi^2}. \tag{55}$$

Now, the Critical wave number is given by

$$a_c=\pi\sqrt{-\frac{1}{2}+\frac{3}{2^{\frac{2}{3}}\left(G+\sqrt{-2916+G}\right)^{\frac{1}{3}}}+\frac{\left(G+\sqrt{-2916+G}\right)^{\frac{1}{3}}}{6(2)^{\frac{1}{3}}}}, \tag{56}$$

Now, the Rayleigh number for the stationary state is given by

$$\left(R_a\right)_S=\frac{\pi^4\Im}{\wp}\left(\frac{\Im^2}{1+\lambda}+\frac{Q}{\pi^2}\right)+R_n\left[L_e-N_A\right]. \tag{57}$$

Where,

$$\Im=\frac{1}{2}+\frac{3}{2^{\frac{2}{3}}\left(G+\sqrt{-2916+G}\right)^{\frac{1}{3}}}+\frac{\left(G+\sqrt{-2916+G}\right)^{\frac{1}{3}}}{6(2)^{\frac{1}{3}}},$$

$$\wp=-\frac{1}{2}+\frac{3}{2^{\frac{2}{3}}\left(G+\sqrt{-2916+G}\right)^{\frac{1}{3}}}+\frac{\left(G+\sqrt{-2916+G}\right)^{\frac{1}{3}}}{6(2)^{\frac{1}{3}}}$$

and G is given by equation (55).

VIII. Validation Of Results

Putting the value of $\lambda = 0$ in the equation (51), we get:

$$R_a=\left(\frac{(\pi^2+a^2)}{a^2}\right)\left((\pi^2+a^2)^2+Q\pi^2\right)-R_n\left(N_A-L_e\right). \tag{58}$$

which is in good argument with the result obtained by Gupta et al. [17].

Without the presence of a magnetic field, i.e., $Q=0$ equation (58) becomes

$$R_a=\frac{(\pi^2+a^2)^3}{a^2}+R_n\left(L_e-N_A\right). \tag{59}$$

Now differentiating equation (59) w.r.t. wave number, we get:

$$\frac{\partial R_c}{\partial a^2}=\left(\frac{\left(\pi^2+a^2\right)^2}{a^2}\right)\left(3-\frac{\left(\pi^2+a^2\right)}{a^2}\right)=0. \tag{60}$$

On solving equation (60), we get $a_c=\frac{\pi}{\sqrt{2}}$ and the Rayleigh number at marginal state, which is given by

$$(R_S)_c=\left(\frac{27\pi^4}{4}\right)+R_n\left(L_e-N_A\right), \tag{61}$$

which is in good argument with the result obtained by Nield and Kuznetsov [13].

When the parameters R_n, N_A and L_e are not present, then equation (61) gives

$$(R_S)_c=\frac{27\pi^4}{4}, \tag{62}$$

equation (62) gives the value of the Rayleigh number at the marginal state obtained by Chandrasekhar [6] for regular fluids.

When $Q=0$ then $G=54$.

Therefore, equation (56) becomes

$$a_c=\pi\sqrt{-\frac{1}{2}+\frac{3}{2^{\frac{2}{3}}\left(54+\sqrt{-2916+54^2}\right)^{\frac{1}{3}}}+\frac{\left(54+\sqrt{-2916+54^2}\right)^{\frac{1}{3}}}{6(2)^{\frac{1}{3}}}}$$

On solving the above equation, we get

$$a_c=\frac{\pi}{\sqrt{2}}.$$

In the absence of a magnetic field and the Jeffrey parameter

$$\left(R_a\right)_S=\frac{\pi^4\left(\frac{1}{2}+\frac{3}{2^{\frac{2}{3}}(54)^{\frac{1}{3}}}+\frac{(54)^{\frac{1}{3}}}{6(2)^{\frac{1}{3}}}\right)}{-\frac{1}{2}+\frac{3}{2^{\frac{2}{3}}(54)^{\frac{1}{3}}}+\frac{(54)^{\frac{1}{3}}}{6(2)^{\frac{1}{3}}}}\left(\frac{\left(\frac{1}{2}+\frac{3}{2^{\frac{2}{3}}(54)^{\frac{1}{3}}}+\frac{(54)^{\frac{1}{3}}}{6(2)^{\frac{1}{3}}}\right)^2}{1+\lambda}\right)+R_n\left(L_e-N_A\right),$$

on solving the above equation, we get

$$\left(R_a\right)_S=\frac{27}{4}\pi^4+R_n\left(L_e-N_A\right). \tag{63}$$

IX. Graphical Result

Fig. 2 visualizes the change of the Rayleigh number $(R_a)_S$ w.r.t. the wave number (a) for non-identical values of the Jeffrey parameter ($\lambda = 0.3, 0.6, 0.9$) under free-free boundary

conditions. It is noticed that, with the increase in the Jeffrey parameter, the value of $(R_a)_S$ decreases, which implies that the marginal state is attained earlier, meaning the onset of convection starts early. Therefore, Jeffrey's parameter (λ) exhibits a destabilizing effect.

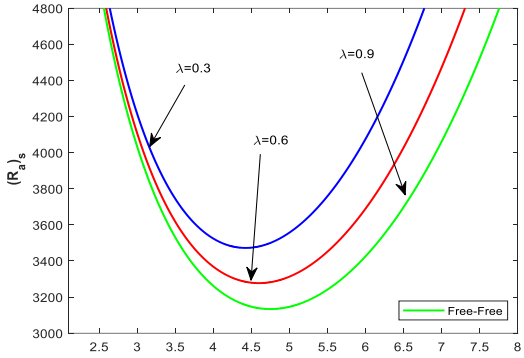


Figure 2. *Graphical Plot between the Rayleigh number* $(R_a)_S$ *and the wave number for different values of* λ.

Fig. 3 displays the change of $(R_a)_S$ w.r.t. a for distinct choices of the Chandrasekhar number (Q = 100, 200, 300) under free-free boundary conditions. It is noticed that a rise in the Chandrasekhar number has a stabilizing influence on the onset of stationary convection, i.e., the Rayleigh number increases with an increase in the Chandrasekhar number. Therefore, the marginal state increases, and convection will be delayed for large values of the Chandrasekhar number.

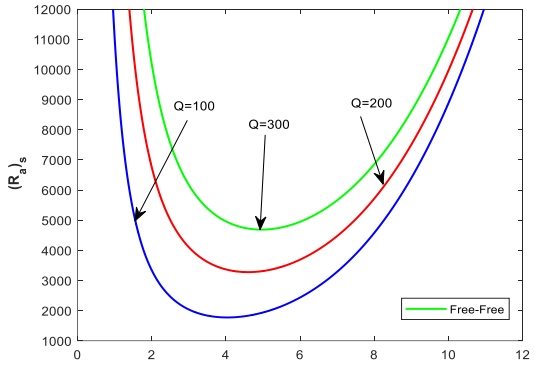


Figure 3. *Graphical plot between* $(R_a)_S$ *and wave number for distinct values of Chandrasekhar number and wave number.*

Fig. 4 illustrates the variation of $(R_a)_S$ w.r.t. a for distinct values of the nanoparticle Rayleigh number (R_n = -1, -0.5, -0.1). It is noted that negative values of R_n have a stabilizing effect. Which means that as the value of R_n increases, $(R_a)_S$ becomes larger than $(R_{a_c})_S$. And convection will be delayed.

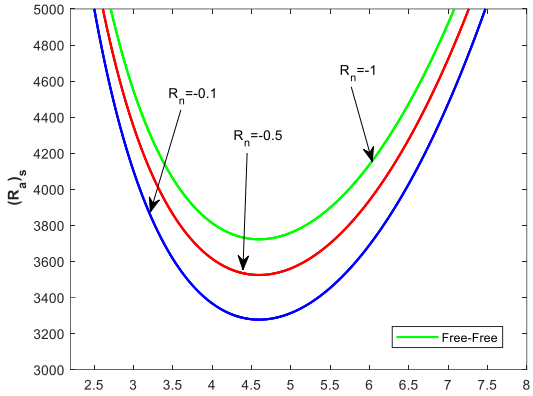


Figure 4*. Graphical representation of* $(R_a)_S$ *with respect to* a *for non-identical values of nanoparticle Rayleigh number.*

Fig. 5 provides the graphical interpretation of the $(R_a)_S$ versus a by taking distinct values of the Lewis number (L_e = 100,500,1000). It is evident that the Lewis number has a destabilizing influence on the onset of stationary convection under both boundary conditions. As the Lewis number increases, $(R_a)_S$ becomes less than $(R_{a_c})_S$; therefore, the convection starts earlier.

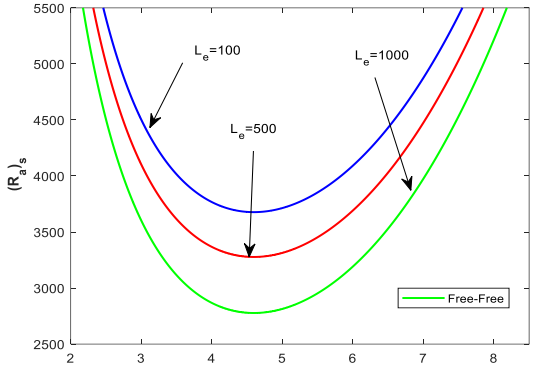


Figure 5. *Visual representation of variation of* $(R_a)_S$ *with* a *by taking distinct values of Lewis number.*

Fig. 6 represents the graph between $(R_a)_S$ and a for non-identical values of the modified diffusivity ratio (N_A = 1, 5, 10) and it is observed that an increase in N_A stabilizes the system and delays the onset of stationary convection. For large values of N_A the system will become more and more stable.

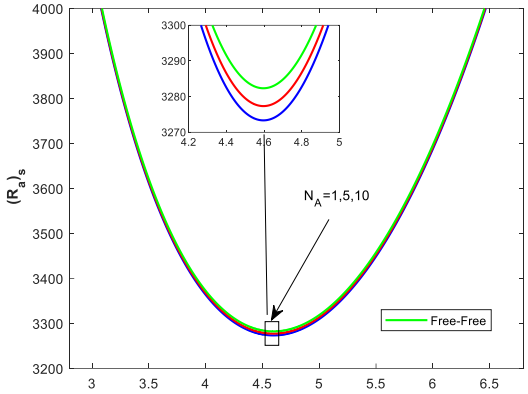


Figure 6. *Graph between* $(R_a)_S$ *vs* a *for unidentical values of modified diffusivity ratio.*

X. Conclusions

The main results obtained about the thermal instability problem of the Jeffrey nanofluid layer when a uniform vertical magnetic field is applied for the case of free-free boundaries are as follows.

- The Jeffrey parameter is showing a destabilizing effect under the effect of a magnetic field.
- The Chandrasekhar number is giving a stabilizing effect on the onset of stationary convection.
- The thermoparticle Rayleigh number for its negative values has a stabilizing effect under the influence of a magnetic field.
- The Lewis number has a destabilizing influence.
- The modified diffusivity ratio has a stabilizing effect on the considered problem.

Author Contributions

P.L. Sharma: Supervision, validation, software, investigation, formal analysis.

T. Thakur: Writing–original draft, conceptualization, methodology, investigation.

S. Gupta: Conceptualization, methodology, writing–review and editing

G.C. Rana: Conceptualization, validation, writing–review and editing

Disclosure Statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] S.U.S. Choi, J.A. Eastman, Enhancing thermal conductivity of fluids with nanoparticles, (No. ANL/MSD/CP-84938; CONF-951135-29). (Argonne National Lab. (ANL), Argonne, IL, United States, (1995).

[2] H. Masuda, A. Ebata, K. Teramae, Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles. Dispersion of Al_2O_3, SiO_2 and TiO_2 ultra-fine particles, Netsu Bussei 7, (1992) pp. 227-233. https://doi.org/10.2963/jjtp.7.227

[3] J.A. Eastman, S.U. Choi, S. Li, W. Yu, L.J. Thompson, Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles, Applied physics letters 78 (2001) pp. 718-720. https://doi.org/10.1063/1.1341218

[4] H., Bénard, Les tourbillons cellulaires dans une nappe liquid, Revue Générale des Sciences Pures et Appliquées 11 (1900) pp. 1261-1271, in French.

[5] L. Rayleigh, LIX. On convection currents in a horizontal layer of fluid, when the higher temperature is on the underside, The London, Edinburgh, and Dublin Phil. Magazine and Journal of Science, 32(192) (1916) pp. 529-546. https://doi.org/10.1080/14786441608635602

[6] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Dover Publication, New York, 1981.

[7] P. Vadasz, Flow and thermal convection in rotating porous media. Handbook of porous media, CRC Press, USA (2000) pp. 395-440.

[8] G.B Jeffery, The motion of ellipsoidal particles immersed in a viscous fluid. Proceedings of the Royal Society of London. Series A, Containing papers of a mathematical and physical character, 102(715) (1922) pp.161-179.

[9] J. Buongiorno, Convective transport in nanofluids, J. Heat and Mass Transfer 128, (2006) pp. 240-250. https://doi.org/10.1115/1.2150834

[10] J. Boussinesq, Théorie analytique de la chaleur: mise en harmonie avec la thermodynamique et avec la théorie mécanique de la lumière (2), Gauthier-Villars (1903), in French.

[11] R.K. Zeytounian, Joseph Boussinesq and his approximation: a contemporary view. Comptes Rendus Mecanique 331(8) (2003) pp.575-586. https://doi.org/10.1016/j.crme.2007.08.002

[12] D. Tzou, Instability of nanofluids in natural convection, J. Heat Transfer 130 (2008) pp. 072401 (9 pages). https://doi.org/10.1115/1.2908427

[13] D. Tzou, Thermal instability of nanofluids in natural convection, Int. J. Heat and Mass Transfer 51 (2008) pp. 2967-2979. https://doi.org/10.1016/j.ijheatmasstransfer.2007.09.014

[14] D.A. Nield, A. Kuznetsov, The onset of convection in a horizontal nanofluid layer of finite depth, European J. Mechanics-B/Fluid 29 (2010) pp. 217-223. https://doi.org/10.1016/j.euromechflu.2010.02.003

[15] D.A. Nield, A.V. Kuznetsov, Thermal instability in a porous medium layer saturated by a nanofluid, Int. J. Heat and Mass Transfer 52 (2009) pp. 5796-5801. https://doi.org/10.1016/j.ijheatmasstransfer.2009.07.023

[16] A.V. Kuznetsov, D. Nield, Thermal instability in a porous medium layer saturated by a nanofluid: Brinkman model, Transp. Por. Media 81 (2010) pp. 409-422. https://doi.org/10.1007/s11242-009-9413-2

[17] A.V. Kuznetsov, D. Nield, The onset of double-diffusive nanofluid convection in a layer of a saturated porous medium, Transp. Por. Media 85 (2010) pp. 941-951. https://doi.org/10.1007/s11242-010-9600-1

[18] U. Gupta, J. Ahuja, R.K. Wanchoo Magneto convection in a nanofluid layer, Int. J. Heat and Mass Transfer 64 (2013) pp. 1163-1171. https://doi.org/10.1016/j.ijheatmasstransfer.2013.05.035

[19] D. Yadav, R. Bhargava, G.S. Agrawal, G.S. Hwang, J. Lee, M.C. Kim, Magneto-convection in a rotating layer of nanofluid, Asia-Pacific J. Chem. Engg. 9 (2014) pp. 663-677. https://doi.org/10.1002/apj.1796

[20] P.L. Sharma, A. Kumar and G.C. Rana, Effect of magnetic field on thermosolutal convection in Jeffrey nanofluid with porous medium. Special Topics & Reviews in Porous Media: An International Journal, 14(3) (2023). https://doi.org/10.1615/SpecialTopicsRevPorousMedia.2023046929

[21] N.Rani, S.K. Tomar, Thermal convection problem of micropolar fluid subjected to hall current, Applied mathematical modelling 34, (2010) pp. 508-519. https://doi.org/10.1016/j.apm.2009.06.007

[22] D. Yadav, R.A. Mohamed, J. Lee, H.H. Cho, Thermal convection in a Kuvshiniski viscoelastic nanofluid saturated porous layer, Ain Shams Engg. J. 8 (2017) pp. 613-621. https://doi.org/10.1016/j.asej.2015.11.023

[23] M. Devi, J. Sharma, U. Gupta, Effect of internal heat source on Darcy-Brinkman convection in a non-Newtonian Casson nanofluid layer, J. Porous Media, 25, (2022) pp. 17-35. https://doi.org/10.1615/JPorMedia.2022039506

[24] R. Chand, D. Yadav, and G.C. Rana, Thermal instability of couple-stress nanofluid with vertical rotation in a porous medium. Journal of Porous Media, 20(7) (2017) pp. 635-648. https://doi.org/10.1615/JPorMedia.v20.i7.50

[25] P.L. Sharma, A. Kumar, G.C. Rana, Effect of magnetic field on thermosolutal convection in Jeffrey nanofluid with porous medium, Spl. Topics & Reviews Por. Media: An Int. J. 14 (2023) pp. 17-29. https://doi.org/10.1615/SpecialTopicsRevPorousMedia.2023046929

[26] D. Bains, P.L. Sharma, G.C. Rana, Darcy-Brinkman magneto-thermal convection in a layer of Casson nanofluid permeated with suspended dust particles, Multiscale and Multidisciplinary Modeling, Experiments and Design 8 (2025) pp. 272. https://doi.org/10.1007/s41939-025-00856-6

[27] P.L. Sharma, P. Lata, A. Kumar, Effect of magnetic field on thermal instability in a porous medium layer saturated by a Jeffrey nanofluid using brinkman model: free-free, rigid-rigid, rigid-free boundary conditions, Structural Integrity and Life 25 (2025) pp. 53-59. http://divk.inovacionicentar.rs/ivk/ivk25/053-IVK12025-PLS-PL-AK.pdf

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