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Research Article

An Innovative Approach to Solve Fuzzy Linear Fractional Programming Problems

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Abstract: Linear Fractional Programming is a mathematical optimization approach that addresses problems involving the optimization of a linear fractional objective function subject to linear constraints. We discussed approach to solving the fuzzy LFPP with and without equality constraints. We have solved this problem without converting it from fuzzy to crisp. First, we changed FLFPP into the FLPP. This problem was converted into parametric form then we solved it using the LU decomposition method to obtain the solution. We presented a numerical example with a real-life application for the simplicity of presenting the algorithm. While most researchers solve FLFPPs using the ranking function method, this method reduces the efficiency of the fuzzy problem. Linear fractional problems with inequality constraints sometimes do not have an optimal solution by using the LU decomposition method. Therefore, we conclude that there is a limitation for this LU decomposition method only for LFP problems with equality constraints.

Keywords: Fuzzy Linear fractional programming; Parametric form; Equality constraint; LU decomposition; Optimal solution.

I. INTRODUCTION

One of the most crucial methods in operations research is the LFPP. LFP is a mathematical optimization technique its applications in corporate planning, hospital planning, production planning, and engineering fields. In LFP, the objective function is a ratio of two linear functions, and the constraints are linear functions. LFPP is a generalization of LPP, which is used to solve optimization problems using linear equations and linear inequalities. While LP computes the best outcome, such as Maximum profit or lowest cost. But LFP is used to achieve the highest ratio of outcome to cost, representing the highest efficiency. Researchers have developed various methods for solving linear fractional problems.

In applications like economics, finance, engineering, and healthcare, exact numerical values are often unavailable due to fluctuating conditions, measurement errors, or human subjectivity. Fuzzy sets provide a flexible mathematical framework to handle such vagueness by representing uncertain parameters as fuzzy numbers. It is a common scenario where the precise values of various parameters are only vaguely known, often represented by fuzzy numerical data, commonly known as fuzzy numbers, this leads to consider the LFPP with fuzzy. In some situations, the utilization optimization is due to its decision criteria. Bellman and Zadeh [1, 2] have contributed to decision-making in fuzzy. Specifically, the concept of decision-making in uncertainty and vagueness. This gives to FLFPP all parameters are denoted as fuzzy numbers. This technique aims at uncertainty and vagueness in the problem, substituting crisp numbers with fuzzy ones. Consequently, the LFP transforms into FLFPP. A pivotal development in FLFPP was introduced by Charles and Cooper [3]. They successfully transformed LFPP to LPP and got solutions using the Simplex method. Fuzzy set theory was discussed in 1991 by Zimmerman [4].

Enables a more realistic representation of uncertain constraints and objective functions. Enhances the flexibility of optimization models in handling practical problems with ambiguous or incomplete information. Provides decision-makers with a range of possible solutions rather than a single deterministic outcome, improving decision flexibility.

Many researchers then used this idea to create numerous methods for solving fuzzy optimization problems. Recent years have seen a rise in interest in fuzzy optimization. Buckley and Feuring [5, 6] solved fully fuzzy LPP with fuzzy numbers. In different situations, researchers have used fuzzy numbers in which just specific aspects of the problem, such as the objective function and right-hand side, or only variables or parameters, were considered to be fuzzy. Ganesan et al. [7] solved FLP problems with symmetric Trapezoidal numbers. Maleki et al. and Safei [8] proposed a method for solving the FLFP problem by using the decomposition method. The FLFP problem was also studied by Pandian et al. [9] using a similar approach. Pop et al. [10] introduced an innovative approach for addressing the full FLFPP with TFN. Their method involves an initial transformation of the fuzzy problem into a multi-objective LFPP accompanied by quadratic constraints. This transformation of the extension principle, notably the Zadeh method. Stanu-Minaian [11] also presented a solution for the fully fuzzified LFPP. Their model begins with the utilization of the Charnes and Cooper method [3] to convert the LFPP into an equivalent LP form. Subsequently, they deal with deterministic multiple-objective LPPs that incorporate quadratic constraints. In this context, it is important to emphasize that the objective functions cost, resource allocations, and technological coefficients are all characterized as TFNs.

Veeramani et al. [12], came up with a new way to solve FLFPP. They divided the problem into smaller parts, each of which is a multi-objective LPP. These smaller problems were solved separately, and this process allowed them to find solution for the original fuzzy LFPP. Murugananthan et al. [13-15] applied the Harmonic technique for solving multi-objective FLFPP and used the LU(Lower-Upper) decomposition method for solving FLFPP converting to the equivalent crisp form [16]. Sadeghi et al. [17] investigated the connection between linear bi-level multi-follower programming problems and multiple objective programming. Kumar et al. [18] solved FFLP problems with equality constraints. Swarup [19] discussed a simple method to solve LFPP. Loganathan et al. [20, 21] solved FLFPP without converting the crisp form. Das et al. [22, 23] suggested a new method to solve the FLFPP and tried FLFP with trapezoidal and [24] suggested Fully FLFPP transform into Fully Fuzzy LPP and again convert to multi-objective LPP to obtain the results. Likewise, fuzzy sets and systems have many real-life applications [25]. Basha et al. [26] solved FMOLPP using the Chandra Sena approach. Gilany et al. [27] introduce a wavelet based fault location method for age cable system using synhronized digital fault data, offering reliable fault detection even near busbars while mitigating the impact of changing cable parameters.

Malik et al. [28] applied a fully intuitionistic fractional programming problem in an E-education system. Kalifa et al. [29] developed MOLFPP with LR variables using a goal program method. Singh et al. [30] scalarised the MOLP by minimum bounded sum operator, gamma connective to get optimality. Risk-Allah et al. [31] Presented a fuzzy TOPSIS and Jaya approach, for solving the BL-MOLFP problem. Peric et al. [32] analyzed MOLFPP for the Iterative method. Das et al. [33] solved Neutrosophic LFPP with mixed constraints and obtained the optimal solution. Bogdana et al. [34] discussed n solution framework that adheres closely to the fundamental principles of fuzzy numbers while addressing certain differences present in a prior approach described in recent literature. Bhatia et al. [35] using the Mehar approach tackle the resolution of fully FLF transportation problems. Farnam et al. [36] focused on hesitant FLFPP, defining various types, extending the Bellman-Zadeh theory for their solution, and proposing a novel approach for one specific type. Bajaj et al. [37] introduce a novel model for solving MOLFPP with triangular intuitionistic fuzzy parameters. It aims to determine acceptable deviations in objective values under constraints, addressing uncertainty through (α, β)-cuts and interval-valued fractional objectives, with a numerical example illustrating the approach. Nayak et al. [38] addressed decision-making uncertainties through fuzzy optimization, focusing on MOLFPP with fuzzy numbers, a solution method involving objective function transformation, constraint centroid derivation, and fuzzy aspiration levels. Borza et al. [39] solved an efficient method for solving FLFPP, α-cuts to handle fuzzy numbers, transforming the problem into a bi-objective LPP through interval arithmetic and transformations of variable. Further simplified into an LPP using max-min theory, proving its optimality for the original fuzzy problem.

Several methods are available for solving FLFPP. α-Cut Method [40] generates a corresponding crisp problem using LPP techniques and [41] used for FLFPP into an equivalent crisp LPP by defining a linear ranking function that assigns weights to the fuzzy numbers. The fuzzy Arithmetic Approach applies to fuzzy numbers in the objective function and constraints, subsequently converting the FLFP problem into a conventional LFPP. Interval-Valued LPP Method is used to convert the FLFP problem into an interval-valued LFPP, which can be solved using existing interval arithmetic techniques. Prasad et al. [52] solved using goal programming approach LFPP. Ali et al. [42] studied the intuitionistic fuzzy MOLPPs under neutrosophic uncertainty. Solving via Metaheuristic Algorithms [43], such as genetic algorithms, particle swarm optimization, and simulated annealing, have been adapted to solve FLFP problems by treating fuzzy numbers as interval-valued or crisp values during the optimization process. Karthick et al. [53, 54, 55] solved trapezoidal LFPP problems. These methods are available for solving FLFPP. For this study, we used the LU decomposition method, which will tackle FLFPPs, offering solutions that balance accuracy and computational efficiency.

Linear Fractional Programming (LFP) has been extensively studied in recent years due to its broad applicability in optimization problems under uncertain environments. The incorporation of fuzzy environments into LFP extends its applicability to real-world scenarios where decision variables and constraints are imprecise. The development of different fuzzy frameworks, such as intuitionistic fuzzy, neutrosophic fuzzy, trapezoidal fuzzy, and triangular fuzzy environments, has significantly enhanced the robustness of optimization techniques [56, 57, 73, 74].

Recent studies have explored various mathematical approaches to solving fuzzy-based optimization problems, such as geometric aggregation techniques, hypersoft sets, and different correlation measures (Bouraima et al. [58], 2025; Ézsiás et al. [59], 2024). These techniques enable researchers to address complex problems in areas including material selection, human capital development, and decision-making under uncertainty (Fatima et al. [60]; Fischer [61]). Furthermore, significant advancements have been made in applying fuzzy decision-making models to domains such as railway track settlement behavior, energy cost reduction in hospitals, and robotic selection (Fischer & Kocsis Szürke [62]; Gökalp & Eti [63]; Imran et al. [64]).

A notable development in fuzzy-based problem-solving techniques is the introduction of fuzzy difference equations and q-rung orthopair fuzzy sets, which have demonstrated improved efficiency in handling uncertainty and imprecision [65, 67, 77, 78]. These methods have also been successfully employed in fields like multi-criteria decision-making and sustainable urban innovation [68-73].

The ongoing advancements in fuzzy optimization methods, including applications of soft computing and intelligent decision analytics, continue to shape the evolution of optimization frameworks [76, 77, 79]. With the continuous development of computational tools and heuristic approaches, researchers are now able to tackle intricate decision-making problems in various industries [69, 75]. This study aims to further contribute to the field by examining the effectiveness of solving Linear Fractional Programming problems under fuzzy constraints and exploring novel methodologies to enhance solution accuracy and computational efficiency.

In this article, we contracted by the following shape: Section 2 we analyzed the FLFPP and their developments based on available literature. Section 3 introduces the concept of fuzzy sets, fuzzy numbers, and Arithmetic Operations. Framing a fractional model for this present study in Section 4. Section 5 explains the flexibility of the adopted methodology and its advantages and also presents an algorithm for FLFPP. We have established the application of this study’s Fractional equality and inequality constraints and we have solved them using the proposed LU-Decomposition-based Technique in Section 6. Finally, the Conclusion of this study is discussed.

II. PRELIMINARIES

Definition 1. [4] If X is a universal set and $x \in X$, then a fuzzy set $\tilde{A}$ defined as, $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x \in X\}$ Where $\mu_{\tilde{A}}$ = membership function.

Definition 2.[20] A fuzzy set $\tilde{A}$ is called fuzzy number if its membership function $\tilde{A}: R \to [0,1]$ satisfies the following conditions:

- $\tilde{A}$ is convex
- $\tilde{A}$ is normal
- $\tilde{A}$ is piecewise continuous.

Definition 3. A fuzzy number $\tilde{A}$ on R is said to be a triangular fuzzy number(TFN)[14] if its membership function $\tilde{A}: R \to [0,1]$ has the following criteria, and this TFN graphical representation is presented in **Fig. 1**.

$$\tilde{A}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, a_1 \le x \le a_2 \\ \dfrac{a_1 - x}{a_3 - a_2}, a_2 < x \le a_3 \\ 0, \text{otherwise.} \end{cases} \quad (1)$$

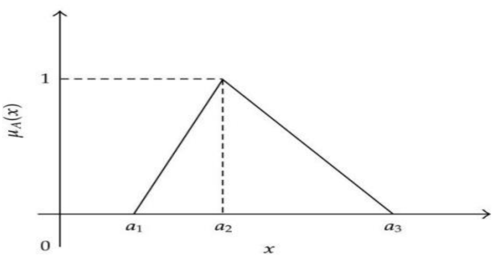

***Figure 1.** Graphical representation of TFN*

The TFN is denoted as notational by $\tilde{A} = (a_1, a_2, a_3)$. and $F(R)$ is used for the set of all TFNs.

Definition 4.[46] A ranking is a function $R\colon\ F(R)\ \rightarrow R$, which maps each fuzzy number into the real line. Let $\tilde{A} = (a_1, a_2, a_3)$ is a TFN then

$$R(\tilde{A}) = \frac{a_1 + 2a_2 + a3}{4} \tag{2}$$

Definition 5. [51] Let $\tilde{A} = (a_1, a_2, a_3)$ be a TFN, then the parametric form defined as

$$\tilde{A} = (a_0, a_*, a^*) \tag{3}$$

where

$$a_* = a_0 - \underline{a} \tag{4}$$

$$a^* = \overline{a} - a_0 . \overline{a}(\chi) = a_3 - (a_3 - a_2)\chi \tag{5}$$

$$\underline{a}(r) = (a_2 - a_1)\chi + a_1 \tag{6}$$

$$a_0 = \frac{\overline{a}(\chi) + \underline{a}(\chi)}{2} \tag{7}$$

where r = 1, we get $a_0 = a_2$ and $\chi \in [0, 1]$.

1. Arithmetic Operation

Performing any two TFNs, Ming Ma et al. [47] was presented parametric arithmetic operations:

Let $\tilde{A} = (a_0, a_*, a^*)$ and $\tilde{C} = (c_0, c_*, c^*)$ be a TFNs then,

$$\tilde{A} * \tilde{C} = (a_0 * c_0, Max\ \{a_*, c_*\}, Max\ \{a^*, c^*\}) \tag{8}$$

where * is Addition $(+)$, Subtraction $(-)$, Multiplication$(\times)$, Division$(\div)$.

III. MODEL FORMULATION

Linear fractional programming is an extension of LPP, the aim of this work is to increase efficient optimization techniques for LFPP to address complex decision-making scenarios where fractional objectives and constraints arise. This approach aims to formulate and solve problems involving rational expressions, contributing to improved resource allocation, cost minimization, and performance optimization in various real-world applications. The objective further involves exploring the algorithm and methodology to enhance the understanding and resolution of the FLFPPs, thus advancing optimization capabilities for practical decision support systems.

The LFPP can be formulated as

$$Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{j=1}^{n} p_j x_j + p_0}{\sum_{j=1}^{n} d_j x_j + d_0} \tag{9}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq \text{or} \geq \text{or} = b_i \tag{10}$$

where $i = 1,2,3, \dots, m$ and $x_j \geq 0$.

FLFPP better than a conventional LFPP by integrating the uncertainty and imprecision inherent to decision-making in the real world. It enables the modeling of vague, qualitative, or uncertain data, enhancing the representation of complex systems. FLFPP offers a more realistic depiction of the fuzzy optimization field, accounting for ambiguity in objectives and constraints. This approach provides a robust framework for addressing practical scenarios where precise numerical values may not adequately represent the complexities of the problem, ultimately resulting in more accurate and trustworthy solutions. The incorporation of fuzzy logic improves the adaptability and flexibility of decision support systems, making them more applicable to real-life problems. A general form of FLFPP with Equality constraint is defined as

$$Max\tilde{z} = \frac{\sum \tilde{a}_i x_i + \tilde{\alpha}}{\sum \tilde{b}_i x_i + \tilde{\beta}} (4) \tag{12}$$

subject to

$$\sum_{i=1}^{n} \tilde{c}_{ij} \tilde{x}_i = \tilde{d}_j \tag{13}$$

where $j = 1,2, \dots, m$ and $x_i \geq 0$ for all $i = 1,2, \dots, n$. The following notations will help to understand the model developments,

m = Number of constraints
n = Number of unknowns
$\tilde{a}, \tilde{b}$ = n-dimensional fuzzy vectors
$\tilde{\alpha}, \tilde{\beta}$ = fuzzy scalars
$\tilde{c}_{ij}$ = $m \times n$ fuzzy cost matrix
$\tilde{d}$ = m dimensional fuzzy vector

Throughout this full paper we considered $\tilde{a}, \tilde{b}, \tilde{c}$ and $\tilde{d}$ are triangular fuzzy numbers.

IV. METHODOLOGY

For consideration, the LU decomposition method in this study and the advantages for solving FLFPP:

- Numerical Stability: LU decomposition enhances numerical stability by decomposing the coefficient matrix into lower and upper triangular matrices. This stability is crucial when dealing with fuzzy numbers, which inherently introduce uncertainty and imprecision.
- Efficiency: LU decomposition reduces the computational complexity of solving systems of linear equations, which are frequently encountered in FLFP problems. Once the

decomposition is performed, subsequent iterations involve forward and backward substitution, leading to faster solution times.

- Simplicity of Implementation: LU decomposition is a well-established and widely used technique in numerical linear algebra. Its straightforward implementation simplifies the process of solving FLFPPs, making it accessible to researchers and practitioners.
- Applicability to Large-Scale Problems: LU decomposition can efficiently handle large-scale FLFPPs, where the coefficient matrix may be sparse or dense. This makes it suitable for real-world applications with complex and extensive fuzzy constraints.
- Integration with Optimization Algorithms: LU decomposition can be seamlessly integrated into various optimization algorithms, allowing for hybrid approaches that combine its advantages with optimization strategies for enhanced accuracy and efficiency in solving FLFPPs.

In conclusion, the LU decomposition method provides numerical stability, computational efficiency, ease of implementation, scalability to large-scale problems, and compatibility with optimisation techniques, making it a useful instrument for solving FLFPPs. The FLFPP conversion into FLPP using Charnes and Cooper method [45] is as follows. Let

$$\tilde{t} = \frac{1}{\sum \tilde{b}_i x_i + \tilde{\beta}} \tag{14}$$

and

$$y_i = \tilde{t} x_i \tag{15}$$

for $i = 1,2,.,n$.

$$Max\tilde{z} = \sum \tilde{a}_i y_i + \tilde{\alpha}\tilde{t} \tag{16}$$

subject to

$$\sum \tilde{b}_i \tilde{y}_i + \tilde{\beta}\tilde{t} = 1 \tag{17}$$

$$\sum_{i=1}^{n} \tilde{c}_{ij} \tilde{y}_i = \tilde{d}_j \tilde{t} (5) \tag{18}$$

where j=1,2,..,m and $\tilde{y}_i, \tilde{t} \geq \tilde{0}$ for all i=1,2,..,n. The expandable form of FLPP is

$$-\sum \tilde{a}_i \tilde{y}_i - \tilde{\alpha}\tilde{t} + \tilde{z} \leq 0 \tag{19}$$

$$\sum \tilde{b}_i \tilde{y}_i + \tilde{\beta}\tilde{t} = 1 \tag{20}$$

$$\sum_{i=1}^{n} \tilde{c}_{ij} \tilde{y}_i \tilde{d}_j \tilde{t} = 0 \tag{21}$$

$$-\tilde{y}_i, -\tilde{t} \leq \tilde{0} (6) \tag{22}$$

The contracted model system has m linear constraints and n unknowns, and then the system is $AY = B$. Where $A = m \times n$ fuzzy matrix. $Y, B = n \times 1$ fuzzy vector.

1. Modified LU-decomposition method

In the context of a system comprising m linear constraints and n unknowns, the following steps outline the LU decomposition method:

- System Formulation: We begin by expressing the system as $AY = B$, where A represents an $m \times n$ fuzzy matrix, Y and B are $n \times 1$ fuzzy vectors. This representation encapsulates the constraints and unknown variables of the problem.
- LU Decomposition: In Step 1, we factorize matrix A into the product of two matrices, L (a unit lower triangular matrix) and U (an upper triangular matrix). This decomposition facilitates the reduction of the original problem into simpler components.
- Matrix Equation Transformation: Upon obtaining L and U in Step 2, we rewrite the system as $LUY = B$. This transformation allows us to work with the decomposed matrices, simplifying the subsequent calculations.
- Intermediate Variable Definition: In Step 3, we introduce an intermediate variable, W, through the equation $UY = W$. This variable helps in breaking down the problem further and isolating components for an easier solution.
- Solution Computation: With W defined, we proceed to solve the equation $LW = B$. By obtaining the values of W, we uncover a crucial aspect of the solution.
- Final Solution: In the last step, we utilize the derived value of W to compute the values of Y, culminating in the solution for the system $AY = B$. This final step ties together the components of LU decomposition, resulting in a comprehensive solution to the given FLFPP.

These steps encompass the LU decomposition approach's progression, starting from the formulation of the problem and culminating in the determination of the solution through a series of well-defined operations on the involved matrices and vectors.

V. ALGORITHM

Step 1: Formulate the FLFPP with equality constraints as the initial framework.

Step 2: Utilize the Charnes and Cooper method to transform the FLFPP into an equivalent FLPP.

Step 3: Convert the FLPP into a parametric form, enabling the introduction of a parameter to facilitate optimization.

Step 4: Employ the LU-Decomposition method, a numerical technique, to efficiently solve the parametric FLPP and obtain its optimal solution.

Step 5: By systematically varying the parameter θ, construct a solution table that provides insights into the behaviour of the optimal solution across different scenarios. This step contributes to a comprehensive understanding of the problem's sensitivity to changes in the parameter value.

We introduced a novel algorithm for solving FLFPP, utilizing the LU (Lower-Upper) decomposition method. This strategy harnesses the inherent numerical stability and computational efficiency of LU decomposition, enhancing solution accuracy while reducing complexities. Our algorithm optimizes the handling of fuzzy constraints and uncertainty inherent in FLFP problems, extending its potential applicability to real-world scenarios. Recent research by Alemohammad et al. [44], Mohammadi et al. [48], Rezaei et al. [49], Song et al. [50] underscore the algorithm's efficacy in addressing FLFP challenges while leveraging the proven foundations of LU decomposition. The methodology employed in this study is above outlined.

For the presented real-life issue in the upcoming section 7. We have solved and analysed the optimal solutions by the proposed algorithm.

VI. APPLICATION PROBLEM

The company engages in the production of two distinct products, namely A and B, each yielding profits of approximately \$5 and \$3 per unit, respectively. However, the costs associated with manufacturing these products stand at roughly \$5 and \$2 per unit, correspondingly. Additionally, a fixed cost of approximately \$1 is incurred. The raw material requisites for producing products A and B amount to roughly three and five units per pound, respectively, subject to a restricted supply of approximately 15 pounds.

Considering the production dynamics, it is established that crafting a unit of product A requires approximately 5 man-hours, while product B entails approximately 2 man-hours per unit. However, the collective availability of man-hours per day is capped at approximately 10 hours. The primary objective here is to ascertain the optimal production quantities of products A and B, thereby Maximizing the overall profit.

Notably, the environmental aspects encompassing profit (reflecting market dynamics), cost (influenced by market), man-powers (linked to workforce efficiency and availability), and raw materials (influenced by wastage) are characterized by imprecise triangular possibility distributions, owing to the presence of incomplete information over the planning horizon. For instance, the profit from product A exhibits a range of (3, 5, 7) dollars, while the man-hours per unit for product A are expressed as (2, 3, 4) hours. Analogously, various other parameters and variables are represented as TFN.

1. Example

In light of these complexities, the problem at hand can be effectively framed as an FFLFPP. To this end, let x_1 and x_2 denote the production quantities of products A and B, respectively. The overarching objective is to devise an optimal solution that navigates the intricate interplay of imprecise environmental coefficients, thereby Maximizing the overall profit under the given constraints.

$$Max\ \tilde{z} = \frac{\tilde{5}\tilde{x}_1 + \tilde{3}\tilde{x}_2}{\tilde{5}\tilde{x}_1 + \tilde{2}\tilde{x}_2 + 1} \tag{23}$$

subject to

$$\tilde{3}\tilde{x}_1 + \tilde{5}\tilde{x}_2 = \tilde{1}5 \tag{24}$$

$$\tilde{5}\tilde{x}_1 + \tilde{2}\tilde{x}_2 = \tilde{1}0 \tag{25}$$

$$\tilde{x}_1, \tilde{x}_2 \geq \tilde{0} \tag{26}$$

The triangular fuzzy numbers are indicated as

$$\tilde{5} = (3,5,7) \tag{27}$$

$$\tilde{3} = (2,3,4) \tag{28}$$

$$\tilde{2} = (1,2,3) \tag{29}$$

$$\tilde{1} = (0,1,2) \tag{30}$$

$$\tilde{1}5 = (11,15,19) \tag{31}$$

$$\tilde{1}0 = (8,10,12) \tag{32}$$

Let $\frac{1}{\tilde{5}\tilde{x}_1 + \tilde{2}\tilde{x}_2 + \tilde{1}} = \tilde{t}$ and $\tilde{t}x_1 = \tilde{y}_1, \tilde{t}x_2 = \tilde{y}_2$. Then the given FLFPP Eq. (23-26) becomes an FLPP as follows by using Charnes and Cooper method

$$Max\ \tilde{z} = \tilde{5}\tilde{y}_1 + \tilde{3}\tilde{y}_2 \tag{33}$$

subject to

$$\tilde{5}\tilde{y}_1 + \tilde{2}\tilde{y}_2 + \tilde{t} = \tilde{1} \tag{34}$$

$$\tilde{3}\tilde{y}_1 + \tilde{5}\tilde{y}_2 - \tilde{1}5\tilde{t} = \tilde{0} \tag{35}$$

$$\tilde{5}\tilde{y}_1 + \tilde{2}\tilde{y}_2 - \tilde{1}0\tilde{t} = \tilde{0} \tag{36}$$

$$\tilde{y}_1, \tilde{y}_2, \tilde{t} \geq \tilde{0} \tag{37}$$

That is

$$Max\ \tilde{z} = (3,5,7)\tilde{y}_1 + (2,3,4)\tilde{y}_2 \tag{38}$$

subject to

$$(3,5,7)\tilde{y}_1 + (1,2,3)\tilde{y}_2 + (0,1,2)\tilde{t} = (0,1,2) \tag{39}$$

$$(1,3,5)\tilde{y}_1 + (3,5,7)\tilde{y}_2 - (11,15,19)\tilde{t} = (0,0,0) \tag{40}$$

$$(3,5,7)\tilde{y}_1 + (0,2,4)\tilde{y}_2 - (8,10,12)\tilde{t} = (0,0,0) \quad (41)$$

$$y_1, y_2, \tilde{t} \geq \tilde{0} \quad (42)$$

In Eq. (38-42) Apply parametric form we get,

$$Max\ \tilde{z} = (5,2-2\chi,2-2\chi)\tilde{y}_1 + (3,1-\chi,1-\chi)\tilde{y}_2 \quad (43)$$

Subject to

$$(5,2-2\chi,2-2\chi)\tilde{y}_1 + (2,1-\chi,1-\chi)\tilde{y}_2 + (1,1-\chi,1-\chi)\tilde{t} = (1,1,1) \quad (44)$$

$$(3,2-2\chi,2-2\chi)\tilde{y}_1 + (5,2-2\chi,2-2\chi)\tilde{y}_2 - (15,4-4\chi,4-4\chi)\tilde{t} = (0,0,0) \quad (45)$$

$$(5,2-2\chi,2-2\chi)\tilde{y}_1 + (2,2-2\chi,2-2\chi)\tilde{y}_2 - (10,2-2\chi,2-2\chi)\tilde{t} = (0,0,0) \quad (46)$$

$$\tilde{y}_1, \tilde{y}_2, \tilde{t} \geq \tilde{0} \quad (47)$$

Since

$$\tilde{Z} \leq Max\ \tilde{Z} \quad (48)$$

$$\tilde{Z} \leq (5,2-2\chi,2-2\chi)y_1 + (3,1-\chi,1-\chi)y_2 \quad (49)$$

The FLP Problem becomes,

$$\tilde{z} - (5,2-2\chi,2-2\chi)\tilde{y}_1 - (3,1-\chi,1-\chi)\tilde{y}_2 \leq (0,0,0) \quad (50)$$

$$(5,2-2\chi,2-2\chi)\tilde{y}_1 + (2,1-\chi,1-\chi)\tilde{y}_2 + (1,1-\chi,1-\chi)\tilde{t} = (1,1,1) \quad (51)$$

$$(3,2-2\chi,2-2\chi)\tilde{y}_1 + (5,2-2\chi,2-2\chi)\tilde{y}_2 - (15,4-4\chi,4-4\chi)\tilde{t} = (0,0,0) \quad (52)$$

$$(5,2-2\chi,2-2\chi)\tilde{y}_1 + (2,2-2\chi,2-2\chi)\tilde{y}_2 - (10,2-2\chi,2-2\chi)\tilde{t} = (0,0,0) \quad (53)$$

$$-\tilde{y}_1, -\tilde{y}_2, -\tilde{t} \leq \tilde{0} \quad (54)$$

We write the system as $AY = B$, where

$$A = \begin{bmatrix} (-5,2-2\chi,2-2\chi) & (-3,1-\chi,1-\chi) & (0,0,0) & (1,0,0) \\ (5,2-2\chi,2-2\chi) & (2,1-\chi,1-\chi) & (1,1-\chi,1-\chi) & (0,0,0) \\ (3,2-2\chi,2-2\chi) & (5,2-2\chi,2-2\chi) & (-15,4-4\chi,4-4\chi) & (0,0,0) \\ (5,2-2\chi,2-2\chi) & (2,2-2\chi,22\chi) & (-10,2-2\chi,2-2\chi) & (0,0,0) \end{bmatrix} \quad (55)$$

$$Y = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{t}\tilde{z} \end{bmatrix} \quad (56)$$

$$B = \begin{bmatrix} (0,0,0) \\ (1,1-\chi,1-\chi) \\ (0,0,0) \\ (0,0,0) \end{bmatrix} \quad (57)$$

We write $A = LU$,

$$A = \begin{bmatrix} (-5,2-2\chi,2-2\chi) & (-3,1-\chi,1-\chi) & (0,0,0) & (1,0,0) \\ (5,2-2\chi,2-2\chi) & (2,1-\chi,1-\chi) & (1,1-\chi,1-1\chi) & (0,0,0) \\ (3,2-2\chi,2-2\chi) & (5,2-2\chi,2-2\chi) & (-15,4-4\chi,4-4\chi) & (0,0,0) \\ (5,2-2\chi,2-2\chi) & (2,2-2\chi,2-2\chi) & (-10,2-2\chi,2-2\chi) & (0,0,0) \end{bmatrix} \quad (58)$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 \\ l_{41} & l_{42} & l_{43} & 1 \end{bmatrix} \quad (59)$$

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ 0 & u_{22} & u_{23} & u_{24} \\ 0 & 0 & u_{33} & u_{34} \\ 0 & 0 & 0 & u_{44} \end{bmatrix} \quad (60)$$

To simplify, we get,

$$U=\begin{bmatrix}(-5,2-2\chi,2-2\chi) & (-3,1-\chi,1-\chi) & (0,0,0) & (1,1-\chi,1-\chi)\\ (0,0,0) & (-1,2-2\chi,2-2\chi) & (1,1-\chi,1-\chi) & (1,2-2\chi,2-2\chi)\\ (0,0,0) & (0,0,0) & (\frac{-59}{5},4-4\chi,4-4\chi) & (\frac{19}{5},2-2\chi,2-2\chi)\\ (0,0,0) & (0,0,0) & (0,0,0) & (\frac{-209}{59},2-2\chi,2-2\chi)\end{bmatrix} \tag{61}$$

$$L=\begin{bmatrix}(1,1-\chi,1-\chi) & (0,0,0) & (0,0,0) & (0,0,0)\\ (-1,2-2\chi,2-2\chi) & (1,1-\chi,1-\chi) & (0,0,0) & (0,0,0)\\ (\frac{-3}{5},2-2\chi,2-2\chi) & (\frac{-16}{5},2-2\chi,2-2\chi) & (1,1-\chi,1-\chi) & (0,0,0)\\ (-1,2-2\chi,2-2\chi) & (1,2-2\chi,2-2\chi) & (\frac{55}{59},2-2\chi,2-2\chi) & (1,1-\chi,1-\chi)\end{bmatrix} \tag{62}$$

Now $LUY=B$. Then we write $LW=B$, where $W=UY$, To solve $LW=B$. We get

$$\begin{bmatrix}(1,1-\chi,1-\chi) & (0,0,0) & (0,0,0) & (0,0,0)\\ (-1,2-2\chi,2-2\chi) & (1,1-\chi,1-\chi) & (0,0,0) & (0,0,0)\\ (\frac{-3}{5},2-2\chi,2-2\chi) & (\frac{-16}{5},2-2\chi,2-2\chi) & (1,1-\chi,1-\chi) & (0,0,0)\\ (-1,2-2\chi,2-2\chi) & (1,2-2\chi,2-2\chi) & (\frac{55}{59},2-2\chi,2-2\chi) & (1,1-\chi,1-\chi)\end{bmatrix}\begin{bmatrix}\widetilde{w}_1\\ \widetilde{w}_2\\ \widetilde{w}_3\\ \widetilde{w}_4\end{bmatrix} = \begin{bmatrix}(0,0,0)\\ (1,1-\chi,1-\chi)\\ (0,0,0)\\ (0,0,0)\end{bmatrix} \tag{63}$$

After simplification, we get

$$\widetilde{w}_1=(0,0,0), \widetilde{w}_2=(1,1-\chi,1-\chi) \tag{64}$$

$$\widetilde{w}_3=\left(\frac{16}{5},2-2\chi,2-2\chi\right) \tag{65}$$

$$\widetilde{w}_4=(\frac{-235}{59},2-2\chi,2-2\chi) \tag{66}$$

We have to solve now $UY=W$,

$$\begin{bmatrix}(-5,2-2\chi,2-2\chi) & (-3,1-\chi,1-\chi) & (0,0,0) & (1,1-\chi,1-\chi)\\ (0,0,0) & (-1,2-2\chi,2-2\chi) & (1,1-\chi,1-\chi) & (1,2-2\chi,2-2\chi)\\ (0,0,0) & (0,0,0) & (\frac{-59}{5},4-4\chi,4-4\chi) & (\frac{19}{5},2-2\chi,2-2\chi)\\ (0,0,0) & (0,0,0) & (0,0,0) & (\frac{-209}{59},2-2\chi,2-2\chi)\end{bmatrix}\begin{bmatrix}\tilde{y}_1\\ \tilde{y}_2\\ \tilde{t}\\ \tilde{z}\end{bmatrix} = \begin{bmatrix}(0,0,0)\\ (1,1-\chi,1-\chi)\\ (\frac{16}{5},2-2\chi,2-2\chi)\\ (\frac{-235}{59},2-2\chi,2-2\chi)\end{bmatrix} \tag{67}$$

To solve this, we will get

$$\tilde{y}_1=(\frac{20}{209},4-4\chi,4-4\chi), \tag{68}$$

$$\tilde{y}_2=(\frac{45}{209},4-4\chi,4-4\chi), \tag{69}$$

$$\tilde{t}=(\frac{1}{11},4-4\chi,4-4\chi), \tag{70}$$

$$\tilde{z}=(\frac{235}{209},4-4\chi,4-4\chi) \tag{71}$$

Therefore,

$$\tilde{x}_1=(\frac{220}{209},4-4\chi,4-4\chi), \tag{72}$$

$$\tilde{x}_2=(\frac{495}{209},4-4\chi,4-4\chi) \tag{73}$$

$$\tilde{z}=(\frac{235}{209},4-4\chi,4-4\chi). \tag{74}$$

For the same problem, Das et al. [22]we obtained a get crisp solutions; moreover same, the crisp solutions are $x_1 = 1.052, x_2 = 2.368,\ MaxZ = 1.124$.

2. Example

Consider the FLFPP with inequality constraints,

$$Max\ \tilde{z} = \frac{\tilde{2}\tilde{x}_1\ +\ \tilde{3}\tilde{x}_2}{\tilde{1}\tilde{x}_1\ +\ \tilde{1}\tilde{x}_2 +\ \tilde{7}} \tag{75}$$

subject to

$$\tilde{3}\tilde{x}_1\ +\ \tilde{1}\tilde{x}_2 \leq\ \tilde{4} \tag{76}$$

$$\tilde{1}\tilde{x}_1\ +\ \tilde{1}\tilde{x}_2 \leq\ \tilde{1} \tag{77}$$

$$\tilde{x}_1, \tilde{x}_2 \geq\ \tilde{0} \tag{78}$$

Let $\frac{1}{\tilde{1}\tilde{x}_1\ +\ \tilde{1}\tilde{x}_2 +\ \tilde{7}} = \tilde{t}$ and $\tilde{t}\tilde{x}_1 = \tilde{y}_1, \tilde{t}\tilde{x}_2 = \tilde{y}_2$. The given FLFPP Eq. (75-78) becomes an FLPP as follows,

$$Max\tilde{z} = \tilde{2}\tilde{y}_1 + \tilde{3}\tilde{y}_2 \tag{79}$$

subject to

$$\tilde{1}\tilde{y}_1 + \tilde{1}\tilde{y}_2 + \tilde{7}\tilde{t} = \tilde{1} \tag{80}$$

$$\tilde{3}\tilde{y}_1 + \tilde{1}\tilde{y}_2 - \tilde{4}\tilde{t} \leq \tilde{0} \tag{81}$$

$$\tilde{1}\tilde{y}_1 + \tilde{1}\tilde{y}_2 - \tilde{1}\tilde{t} \leq \tilde{0} \tag{82}$$

$$\tilde{y}_1, \tilde{y}_2, \tilde{t} \geq\ \tilde{0} \tag{83}$$

That is

$$Max\tilde{z} = (1,2,3)\tilde{y}_1 + (1,3,5)\tilde{y}_2 \tag{84}$$

subject to

$$(0,1,2)\tilde{y}_1 + (0,1,2)\tilde{y}_2 + (5,7,9)\tilde{t} = (1,1,1) \tag{85}$$

$$(1,3,5)\tilde{y}_1 + (0,1,2)\tilde{y}_2 -\ (2,4,6)\tilde{t} \leq (0,0,0) \tag{86}$$

$$(0,1,2)\tilde{y}_1 + (0,1,2)\tilde{y}_2 - (0,1,2)\tilde{t} \leq (0,0,0) \tag{87}$$

$$\tilde{y}_1, \tilde{y}_2, \tilde{t} \geq\ \tilde{0} \tag{88}$$

By the same way we solving we obtain the values

$$\tilde{x}_1 = (\frac{3}{2}, 2 - 2\chi, 2 - 2\chi), \tag{89}$$

$$\tilde{x}_2 = (\frac{-1}{2}, 2 - 2\chi, 2 - 2\chi) \tag{90}$$

$$\tilde{z} = (\frac{3}{16}, 2 - 2\chi, 2 - 2\chi). \tag{91}$$

Likewise, we have changed the values ofχ from 0 to 1. The changes in how this results in reaching an optimality of this defined problem has formulated It will give more clearance to this research.

The adopted problem [6] was already solved in a crisp environment; the crisp solutions by that study are $x_1 = 0, x_2 = 1, MaxZ = \frac{3}{8}(0.375)$. From the observation of the above 1. and 2. examples by this study, our proposed method producing fuzzy solutions is more differs from the crisp solutions. Therefore, our proposed LU decomposition Method attains the optimal solution for the FLFPP with equality constraints but does not attain the optimal solution for the FLFPP with inequality constraints.

VII. CONCLUSION

In this study, we successfully addressed the problem of FLFPP with equality constraints by employing the LU decomposition-based method. The utilization of the parametric form empowers decision-makers to discern and opt for a favoured solution, a distinct advantage not readily achievable through alternative approaches. The presented numerical example highlights the method's inherent simplicity and ease of implementation. Notably, the LU Factorization-based method demands fewer computations compared to conventional techniques, rendering it a more streamlined and expedient approach than the simplex method. In the course of this research, we tackled two distinct problems. In the context of equality constraints, our methodology yielded an efficient solution, underscoring its efficacy. However, when confronted with inequality constraints, the solution may not invariably achieve optimality. Looking ahead, our future endeavours will encompass addressing FLFP cases involving inequality constraints, with the aim of demonstrating the attainment of efficient solutions within this extended framework.

AUTHOR CONTRIBUTIONS

Karthick Sivakumar: Conceptualization, Experiments.

Saraswathi Appasamy: Theoretical analysis.

Dragan Pamucar: Verification, Supervision.

DISCLOSURE STATEMENT

The authors declare no conflicts of interest.

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