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Research Article

Sawmill scheduling: an application-oriented model

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Abstract: Sawmills play a key role in the primary sector of the wood industry producing lumber from tree logs through a multi-stage process including inspections, classification, sawing, drying, etc. The sawing step can become a bottleneck due to the high investment costs the necessary equipment present. Thus, their schedule can be of significant economic importance, resulting in several studies over the years. While most of the approaches in the literature consider a simple model of production and focus on the stochastic nature of real-life problems, present work details a more in-depth model to better tackle practical considerations of less automated and smaller sawmills. The proposed Mixed-Integer Linear Programming model addresses volatile labour availability and differences between the two most dominant sawing technologies. The efficiency of the model is tested on randomly generated instances. The proposed approach can provide the optimal solution within reasonable time for short-term instances.

Keywords: sawmill; scheduling; optimization; timber industry

I. Introduction and Literature Review

As sustainability becomes an increasingly pressing issue, the application of renewable materials gains ever growing focus. Wood is one of nature's CO2 sequestration technologies, and the timber industry plays a key role in producing products that can store the captured carbon in the long-term. However, the widespread application of wood-based products requires not only feasible technologies and sufficient quality. The whole value chain of these products must be financially competitive to that of other non-renewable options such as plastic. As a result, improving the efficiency of the supply chain and each individual facility can be vital to real-life adaptation.

While many works in the literature focus on high-level optimization of supply chains in the wood industry [1, 2, 3], present work addresses the optimal operation of sawmills that play a key role in the primary wood industry. Sawmills are responsible to produce various primary wood products such as lumber and plank from unprocessed wood logs. The production process is a complex sequence of steps from inspection and classification via the actual sawing to various treatments as shown **Fig. 1**. Sawing equipment present a significant cost of investment in such facilities, thus, their optimized utilization is of importance for the overall efficiency of the plant.

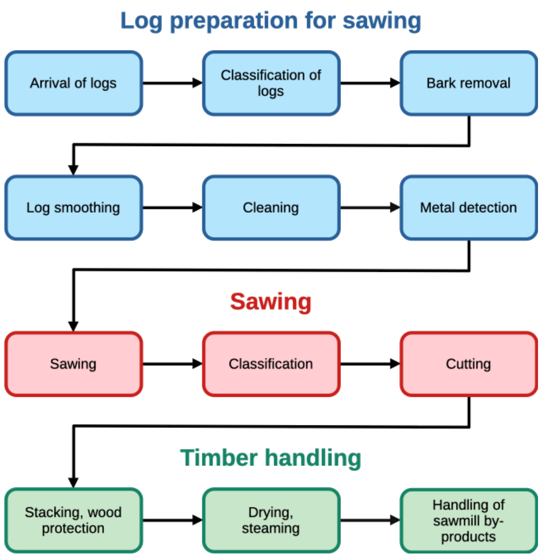

***Figure 1.** Sawmill production process*

The decision flexibility of the production planner is the selection of log classes to be processed and the applied cutting pattern for the span of the entire time horizon. Optimization of cutting pattern is another well-researched field [4] whose goal is generally to minimize waste. At this stage of production planning, it is assumed that the list of cutting patterns to apply are determined a-priori. Each pattern is applicable to a different log class with a specific range of diameter and yields different quantities of various products. An example for such a cutting pattern is shown in **Fig. 2**.

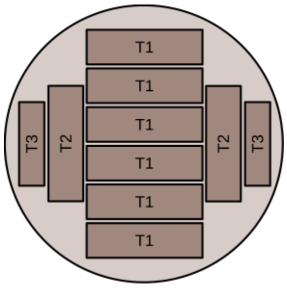

Figure 2. *Cutting pattern example*

Cost-optimal planning of the sawing step was first examined by [5], where the time horizon was split to intervals and subintervals. For each subinterval, the model could determine the portion of that time allocated for each cutting pattern. The objective was to minimize the cost consisting of both late or missed deliveries and storage. The presented model considered only a single cutting pattern for each log class, a single saw, and deterministic behaviour. This model has been extended and generalized many times in the literature. In [6] and [7] the possibility for deciding the cutting pattern for the logs with the same diameter were introduced, while [8] extended the model to address multiple sawing equipment. Another direction of research focused on incorporating non-deterministic behaviour into the optimization process, mostly via robust scheduling as presented in [9] and [10].

All of these models, however, consider a simplified, high-level model of the sawing procedure itself, as a single-stage continuous process. This estimation is adequate for high level planning, and precise enough for highly automated sawmills with huge annual throughput. However, as the scale of the facility decreases, technological details become more prevalent, providing the motivation for the proposed investigation and model of this work.

II. Motivation and Problem Definition

Sawmills show vast variety in both scale and the age or modernity of equipment. Large-scale facilities tend to be highly automated requiring minimal human interventions for either its operation or material movement. On the other end of the spectrum, family-owned small businesses tend to have older equipment whose operation requires more human work both for moving the logs and lumbers and for the operation of saws. Moreover, special domain specific knowledge is needed for issues like the ideal positioning of logs, avoiding defects, etc., which are overcome by cutting-edge sensors and actuators at their modernized counterparts. From the modelling point of view, high-volume facilities with modern equipment can be adequately addressed as a single-stage continuous process, sometimes with optional setup times. For this case, simple linear scheduling models can provide sufficient accuracy. For small-scale facilities, many assumptions of these models are not met. In this work we address some of these, as a first step towards a more process specific model.

One key domain specific feature not addressed in literature models is the basic technology used for sawing. While new techniques are being developed at several companies, often mixing previous ideas and new innovations in sensor technology, robotics, and artificial intelligence [8, 9], most existing equipment can be categorized into two major groups: frame saws and band saws. Frame saws have been around for a long time and are ideal when many logs of the same size are to be sawed with a plain cutting pattern. The saw operates several bands in parallel, thus, a single pass produces all the lumbers from the log. In contrast, band saws cut lumbers in a layer-by-layer fashion in each pass. This operational difference influences several parameters of these family of saws, that are relevant from the scheduling point of view. While each specific brand of equipment has their own specifications, in general, framesaws tend to be faster, cheaper to operate. However, the bands must be thicker resulting in more waste, and changing the cutting pattern requires non-negligible time. On the other hand, band saws can switch easily from one cutting pattern to another, moreover, if deficiencies in the log are detected (either by sensors or by experts operating the saw), agile decisions can be made mid-process to ensure quality products and minimize waste.

Another aspect often disregarded by literature models is the human resource requirement of the entire process, that has a wide range for the various equipment available. Highly automated modern equipment tends to require less human intervention from a highly trained employee. Facilities with large throughput and thus, revenue, tend to have less difficulty securing and training this smaller but specialized workforce. On the other hand, smaller facilities with older equipment require more workers for moving the logs, lumber and operating the saw. While the actual numbers depend on the specific unit, the difference between framesaws and bandsaws may also show in this segment, the former tending to require more people. The human-resource aspect highlights another issue that is more prevalent in smaller businesses: smaller production scale leads to less revenue, which implies smaller budget for investment, thus equipment is older and less modernized, requiring more employees with basic skillset. In the current workforce market, these companies with lower budget often have difficulty securing the additional operators needed for this older equipment. Shortage of such workforce is becoming a pressing issue for such companies.

The aim of this work is to provide an approach, that takes into account several of the aforementioned aspects of the sawmill industry, in order to better model its specific features, and provide more meaningful results. There are many productions specific details not yet addressed here, such as the maintenance frequency and cost, moisture content, and its effects, etc. The goal of this work is to make the first step in the direction of a very specific and detailed model, while evaluating the cost in complexity and computational time needed to solve it.

The overall objective is to minimize cost associated with under-delivering for the accepted orders. The facility has both a framesaw and a bandsaw to utilize. Each equipment has its human resource requirement, and the employees are categorized into two groups: specialists, from whom at least one is needed for the operation of both equipment, and assistant workers – referred to workers later – for tasks like log and lumber movements, etc.

For each day of the planning horizon given are the available workers and specialists, the quantity in each lumber type to deliver. The facility has limited storage capacity for overproduction and there is a limited budget for daily hires as workers. Specialists may also substitute as workers if only one of the saws are operated.

The bandsaw may change cutting patterns any number of times during the shifts, however, framesaws are only allowed to do that once in a single shift. This limitation stems from industrial best practice. The cutting pattern of the framesaw may also be changed on its idle days.

Logs are presumed to be sorted into a finite number of classes, each having one or several cutting pattern, yielding predetermined ratio in a finite number of lumber types. Logs are presumed to be readily available and the beginning of the planning horizon, and of the same wood type.

The production planner has the flexibility to decide:

- On which days are additional hires employed.
- Which equipment is operated on each day.
- If the bandsaw is operated, how long each cutting pattern is used during the shift.
- If the framesaw is operated, what cutting pattern is used, and whether there is a change in the cutting pattern during the day. If so, when, and what is the new cutting pattern.
- The selection of the new cutting pattern if the framesaw is not operated.

III. PROPOSED MODEL

We propose a Mixed-Integer Linear Programming (MILP) model to provide the optimal short- or mid-term schedule of the sawmill under investigation. The objective function can be summarized as follows:

$$\min \sum_{d \in \mathcal{D}} \sum_{t \in \mathcal{T}} LF_t \cdot u(p)_{d,t} \tag{1}$$

Where LF_t is the proportional late fee for lumber type t, and $u(p)_{d,t}$ is the underproduction for that lumber type on day d. Over- and underproduction for a day is managed by the following constraint:

$$y_{d,t} + o(p)_{d-1,t} - o(p)_{d,t} + u(p)_{d,t} \geq DM_{t,d} \quad \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{2}$$

Where $y_{d,t}$ denotes the daily production, $DM_{t,d}$ the delivery requirement. $o(p)_{d-1,t}$ is the overproduction from the previous day, that equals to the amount stored. Similarly, $o(p)_{d,t}$ represents the amount to be stored for the next day. This quantity is limited by overall storage capacity of the facility denoted by C, and shared by all lumber types, as expressed in Equation (3):

$$\sum_{t \in \mathcal{T}} o(p)_{d,t} \leq C \quad \forall d \in \mathcal{D} \tag{3}$$

Daily production quantity is calculated based on Equation (4):

$$y_{d,t} = \sum_{p \in \mathcal{P}} Y_{t,p} \cdot V_{l_p} \cdot \left(q^{F,-}_{d,p} + q^{F,+}_{d,p} + q^{B}_{d,p}\right) \quad \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{4}$$

Where V_{l_p} indicates the volume of a log type l_p of the cutting pattern p, the parameter $Y_{t,p}$ expresses the ratio of the volume of lumber type t produced when using pattern p. Integer variables $q^{F,-}_{d,p}$, $q^{F,+}_{d,p}$, and $q^{B}_{d,p}$ indicate the number of logs cut by pattern p on day d by the frame saw before pattern change, after pattern change, and by the band saw respectively.

Naturally, the number of logs processed by both saws during the whole production horizon cannot exceed the stock available, denoted by I_l for each log type 1, as expressed in Equation (5):

$$\sum_{d\in\mathcal{D}}\sum_{\substack{p\in\mathcal{P}\\ l_p=l}}\left(q_{d,p}^{F,-}+q_{d,p}^{F,+}+q_{d,p}^{B}\right)\le I_l \quad \forall l\in\mathcal{L} \tag{5}$$

The number of logs processed by the band saw is limited by the length of the shift H, and the sawing time ST_p^B for pattern p. Moreover, if the bandsaw is idle on day d, as indicated by a 0 value of the binary variable w_d^B, the overall production is zero.

$$\sum_{p\in\mathcal{P}} q_{d,p}^{B}\cdot ST_p^B \le H\cdot w_d^B \quad \forall d\in\mathcal{D} \tag{6}$$

Modelling the specific behaviour of the framesaw requires several constraints:

$$q_{d,p}^{F,-}\le \frac{t_d}{ST_p^F}+\frac{H}{ST_p^F}\cdot\left(1-s_{d,p}^{F,-}\right) \quad \forall d\in\mathcal{D},p\in\mathcal{P} \tag{7}$$

$$q_{d,p}^{F,-}\le \frac{H}{ST_p^F}\cdot s_{d,p}^{F,-} \quad \forall d\in\mathcal{D},p\in\mathcal{P} \tag{8}$$

$$q_{d,p}^{F,+}\le \frac{H-t_d-CT^F\cdot\left(1-s_{d,p}^{F,-}\right)}{ST_p^F} +\frac{H}{ST_p^F}\cdot\left(1-s_{d,p}^{F,+}\right) \quad \forall d\in\mathcal{D},p\in\mathcal{P} \tag{9}$$

$$q_{d,p}^{F,+}\le \frac{H}{ST_p^F}\cdot s_{d,p}^{F,+} \quad \forall d\in\mathcal{D},\ \mathrm{p}\in\mathcal{P} \tag{10}$$

In these constraints, continuous non-negative variable t_d indicates the time of changing the cutting pattern during the shift, if such an event occurs. Binary variables $s_{d,p}^{F,-}$ and $s_{d,p}^{F,+}$ indicate if pattern p is used before or after the change, respectively. Equations (7) and (8) express the relation $q_{d,p}^{F,-}\le t_d\cdot\frac{s_{d,p}^{F,-}}{ST_p^F}$ in a linear form, where ST_p^F is the time needed to saw a log with pattern p on the framesaw. Equations (9) and (10) express the same relation after the change of the cutting pattern, excluding the changeover time for the new pattern, CT^F.

Similar to the bandsaw, if the framesaw is idle on day d, no logs can be processed by it, as expressed by Equations (11) and (12), where binary variable w_d^F indicates if the framesaw is operated on day d or not:

$$\sum_{p\in\mathcal{P}} s_{d,p}^{F,-}=w_d^F \quad \forall d\in\mathcal{D} \tag{11}$$

$$\sum_{p\in\mathcal{P}} s_{d,p}^{F,+}=w_d^F \quad \forall d\in\mathcal{D} \tag{12}$$

The persistence of the same cutting pattern on subsequent non-idle days is expressed by constraints (13) and (14):

$$s_{d,p}^{F,+}\ge s_{d+1,p}^{F,-}-1\cdot\left(2-w_d^F-w_{d+1}^F\right) \quad \forall d\in\mathcal{D},p\in\mathcal{P} \tag{13}$$

$$s_{d,p}^{F,+}\le s_{d+1,p}^{F,-}+1\cdot\left(2-w_d^F-w_{d+1}^F\right) \quad \forall d\in\mathcal{D},p\in\mathcal{P} \tag{14}$$

The following constraints address the human resource requirements posed by both sawmills:

$$RR^{SP,F}\cdot w_d^F+RR^{SP,B}\cdot w_d^B\le HR_d^{SP} \quad \forall d\in\mathcal{D} \tag{15}$$

$$RR^{AW,F}\cdot w_d^F+RR^{AW,B}\cdot w_d^B\le HR_d^{AW}+x_d +\left(HR_d^{SP}-RR^{SP,F}\cdot w_d^F-RR^{SP,B}\cdot w_d^B\right) \quad \forall d\in\mathcal{D} \tag{16}$$

Where integer parameters RR represent the number of workers (AW) and specialists (SP) for the framesaw (F) and bandsaw (B). Parameters HR_d^{SP} and HR_d^{AW} indicate the daily availability of specialists and workers, respectively. The non-negative integer variable x_d denotes the number of additional hires for day d, who can only be employed in the role of workers, and Equation (16) also accounts for specialists substituting as workers.

Finally, the number of additional hires cannot exceed the predetermined number, as expressed in Equation (17):

$$\sum_{d\in\mathcal{D}} x_d\le X^{AW} \tag{17}$$

IV. EMPIRICAL RESULTS

The model proposed in Section III need to be investigated from two different aspects: practical applicability and efficiency. Thus, two different investigations are presented in this section. First, an example based on data from domain experts is investigated in more detail with special attention on the effect of workforce limitations. Second, computational performance is evaluated on a large number of randomly generated instances.

The implemented model was solved by Gurobi Optimizer 10.0.1 on a computer with an Apple M2 CPU and 16 GB of RAM available.

1. Practical investigation and preliminary efficiency tests

For the practical tests, 3 log sizes and 12 different lumber types were considered. The yield of the 5 cutting patterns were calculated with Pitago Optimizers [13] software. The time required to cut a single log on each machine for each cutting pattern was determined based on consultation with field experts, just as the changeover-time for the framesaw. The planning period was 1 week with a total number of 5 shifts of 8 hours. As stated before, both saws require one specialist, and for the scope of this study it was assumed that an additional worker is needed for the bandsaw while 3 more is required for the framesaw. The availability of the employees for this study are given in **Table 1** for the planning horizon:

Table 1. *Availability of employees*

Day	*Specialists*	*Assistant workers*
1	2	1
2	2	1
3	1	3
4	2	1
5	1	3

The total demand was set for 366 m^3, distributed among the days with 20 m^3 of storage capacity. The lateness cost LF_t for each lumber was set homogeneously to 1 cu/m^3, thus the objective is equivalent to minimizing undelivered quantity.

With no budget for additional hires the model was solved to optimality in 0.8328 seconds, and a schedule that could satisfy 38.19% of the demand. This schedule is shown on the first Gantt diagram in **Fig. 3**, and utilizes only the band saw, frequenting cutting patterns C and E.

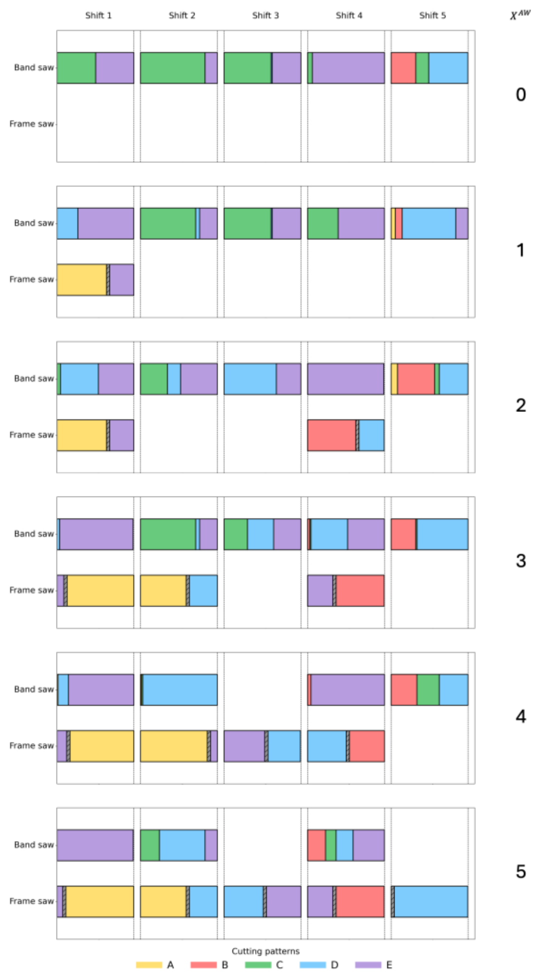

Figure 3. *Gantt chart with 0 additional workers*

In order to test the effect of potential additional hires, the model was solved repeatedly while incrementing the value of X^{AW}. As shown in **Fig. 4**, additional workers managed to increase the production to reduce the unmet demand. The first additional hire allowed the operation of the frame saw on the first shift, introducing cutting pattern A to the schedule, and reducing the unmet demand from by 17.29%. The introduction of a second additional hire reduced the optimum by a similar magnitude, 19.14%. This additional hire was utilized on the 4th shift, allowing the frame saw to operate on that day too. Interestingly, the prevalence of the cutting pattern C diminished, while B became favoured on both saws. The third addition provided a smaller effect of 13.07% and allowed the frame saw to operate on the second day too. The fourth additional hire shows the first schedule, where the band saw is idle on one of the days. The same change can be observed by allowing a 5th hire. In this schedule the frame saw is working every day, while the band saw is idle for 2 days. The unmet demand is reduced to 16.25%. The Gantt diagrams of the optimal schedules for each X^{AW} value can be seen in **Fig. 3**. Any further increase in the temporary workforce did not result in economic benefit. The sensitivity analysis on the diagram in **Fig. 4** can be a valuable tool for the production planner to determine whether additional hires are worth investing into or not.

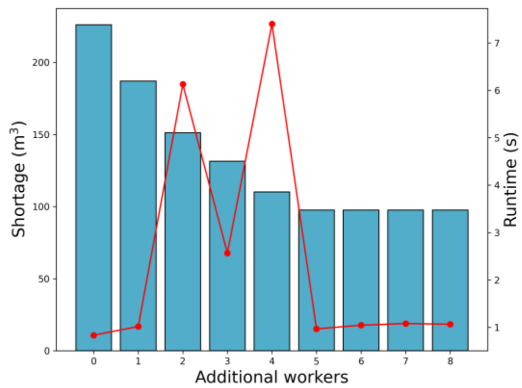

Figure 4. *Result of additional worker increase*

Fig. 4 also shows the CPU time needed to solve the model. It can be seen from the data, that the option for additional hires also increases the search space, thus the computational need as well. However, when no additional benefit can be gained by additional hires, no computational costs are paid for that flexibility either.

2. Computational tests on randomly generated examples

For a more extensive computational test, 6 groups of 50 examples were randomly generated and solved for planning periods ranging from 1 to 6 weeks. For these 300 test cases, the technical parameters of the facility remained the same, and no additional hires were allowed. Within in each instance group corresponding to a planning period, the initial stock size and the cumulative demand of all lumber types were fixed, only the distribution of the demand and staff availability were altered. The stock size and cumulative demand was set proportional to the length of the corresponding planning period. For the instances of 1 week of planning period, 250 m^3 of initial stock of each log type and a total of 400 m^3 of demand was set.

In each test run, the time limit for the optimizer was set to 1500 seconds, which was only reached for outlier cases as shown on the box plot diagram with an exponential y scale in **Fig. 5**.

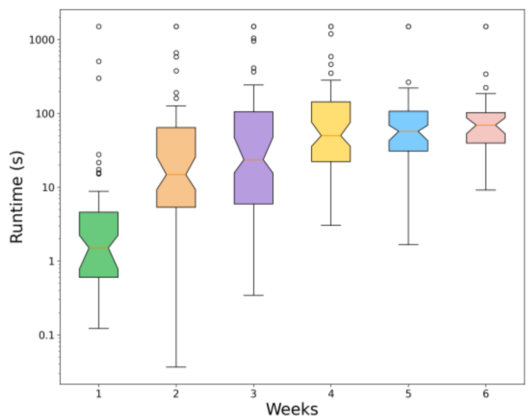

Figure 5. *Computational need of solving the proposed model on randomly generated instances*

The computational tests showed a surprising result. As the number of binary variables in the models increased linearly for each extension of the planning horizon, the required CPU time to solve the instances was expected to increase exponentially. This behaviour can be observed between the instance groups of 1 and 2 weeks of planning horizon. While later increases in the time horizon still resulted in larger CPU times, the change is less drastic, and by week 5 this seems to flatten out. Much larger examples for long-term planning would probably still become unsolvable in a reasonable amount of time. However, practical significance of short- and mid-term planning is much higher, thus these tests focused on planning periods up to one and a half months, where the proposed model proved to be applicable. This also means, that there is potential room to include further practical details while keeping computational needs on a reasonable level.

V. Concluding Remarks

We developed a MILP formulation for the scheduling of smaller sawmills whose novelty lies in addressing several domain specific features of this production process, such as the different parameters of framesaws and bandsaws, and the workforce challenges facing the industry. The aim of integrating these features into the mathematical model responsible for production planning was to provide solutions that adhere to these practical considerations instead of risking sub-optimality by adjusting the schedule in a later phase of the decision-making process. The sensitivity analysis on the workforce availability showed the significance of these parameters on the optimal schedule, underlining the importance of its integration. The computational efficiency of the model has been tested on numerous examples and proved to be a suitable tool for short- and mid-term problems, that enables further research towards more detailed and complex models.

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AUTHOR CONTRIBUTIONS

Csaba Kebelei: Problem definition, model implementation, testing, writing, editing.

Mate Hegyhati: Problem definition, model development, writing.

Disclosure Statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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