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Research Article

Parametric analysis of the stress-strain state for the unsupported and supported horizontal underground workings

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Abstract: An analysis of analytical and numerical approaches to the problem of determining the stress-strain state of underground workings has been carried out. A system of parametric analysis of the stress-strain state of unsupported and supported horizontal underground workings has been developed. This system proved that to determine the stress-strain state of horizontal unsupported workings of a certain diameter, it is enough to perform one numerical calculation of a finite-element model with unit parameters, and using simple analytical formulas, extrapolate the stress-strain state of this model to all possible cases of real workings. Finite element models of underground workings were developed to justify the author's system. A method of parametric analysis of the strained state of a supported working by using models with equivalent bending stiffness has been developed and substantiated. On its basis, it is possible to perform a numerical analysis of a finite-element model with unit parameters of the frame, selecting the equivalent modulus of elasticity.

Keywords: *finite element modelling; horizontal underground working; parametric analysis; rock massif; stress-strain state*

I. INTRODUCTION

In geomechanics, two conceptual directions in the research of the stress-strain state of horizontal underground workings have been formed and are further developing: 1. It is based on an analytical approach; 2. It is based on numerical methods. Reviews in the application of these directions conducted by many researchers [1-3] allow us to highlight their inherent features, wherein these features are most often the advantages and disadvantages of calculation methods.

For example, methods based on an analytical approach are noted by the universality of solutions that can be applied to different conditions, but their disadvantage is a significant number of assumptions introduced when obtaining [3, 4]. Numerical methods, in turn, are characterized by the possibility of calculating many complex underground facilities [5-7]. But their disadvantage is that the obtained solutions refer to specific calculation cases and cannot be extrapolated to similar cases with changed properties [1, 3, 6].

Recently, analytical methods have been much less often used in the calculations of underground facilities, which are explained by the complexity of their application (complicated mathematical apparatus, insufficient software implementation, etc.). A weighty argument for applying numerical methods is a specific approach to each underground facility. This approach is more expedient and rational than a typical calculation without taking into account the specific characteristics of the behavior of an underground structure.

The finite element method is the most widely used numerical method today [3, 5-9]. Its large-scale application is explained by the development of theoretical foundations, the simplicity of algorithmization, and the availability of powerful professional calculation complexes. However, at the same time, in applying the finite element method in the calculations of underground facilities, including horizontal workings, there was a gradual separation from the methodology developed by analytical methods, which led to some one-sided research in this direction [8-11]. It is due to the finite element method's specific features since obtaining high-accuracy solutions; the adequate finite-element model should be developed for real conditions. The mentioned feature of the finite element method (the impossibility of extrapolating the numerical solution of a specific problem to another one with somewhat changed characteristics) is proclaimed by some authors who tend towards an analytical approach as a methodological drawback. However, obtaining a sufficiently accurate solution for a specific calculation case is sometimes more important than obtaining a dependence that can be applied to other cases.

Undoubtedly, the knowledge of the mechanisms for forming the stress-strain state in the “horizontal working – rock massif” system is a key issue of geomechanics. The currently existing methods in studying the stress-strain state are a set of disembodied methods for calculating individual components of stresses and strains, which can be applied only in limited research conditions. Such a situation, in many cases, does not allow obtaining a quantitative picture of the distribution of the stress-strain state in the “horizontal working – rock massif” system. Therefore, a new methodological technique is needed to solve this problem, which allows performing operational determination of the stress-strain state for horizontal working.

II. Methods

Such a methodological technique is parametric analysis developed by the authors based on numerical analysis using the finite element method [3, 8, 9]. Within the framework of this article, the parametric analysis as the research of the stress-strain state of a horizontal working is understood, during which a specific numerical solution is extrapolated to other calculation cases using the author’s algorithms. Exactly these algorithms help solve the issue of the imaginary impossibility of distributing a numerical solution, which is declared by authors who tend toward clear analytical solutions. However, the parametric analysis does not break links with the analytical approach [10, 11].

It is known that representatives of the analytical approach often ground their solutions for horizontal underground workings on the theoretical patterns of Kolosov-Mushkelishvili’s theory of functions of a complex variable, that is, one of the theories of continuum mechanics [3, 7]. In practice, this theory is implemented using conformal mappings, the essence of which is as follows. The real working of any delineation with a defined specific radius R by means of direct mapping turns into a circular working with a unit radius, to which the patterns of Kolosov-Mushkelishvili’s theory are applied. Then, after obtaining the solution of the stress-strain state on a circular working with a unit radius, it is extrapolated to a real working with a defined specific radius R by means of inverse conformal mapping.

A critical analysis of this approach proves that a powerful mathematical apparatus is needed to obtain them, which is most often not implemented in software complexes. However, the very conceptual procedure of Kolosov-Mushkelishvili’s theory is fruitful. Its reinterpreting in line with numerical solutions is that the stress-strain state on a circular working with a unit radius and with the help of special parameters can be extrapolated to other workings. Thus, it is possible to obtain a solution for the stress-strain state on the working with a unit radius without applying complex direct and inverse conformal mappings by scaling the properties of the system [10, 11].

Unquestionably, the application of scaling for unsupported and supported workings with some similarities in the methodology still differs in both theoretical and practical terms. This is an objective position, since the presence of a frame, which is constructed in an unsupported working, radically changes the stress-strain state in the “horizontal working – rock massif” system. The engineering structure/the rock massif interaction is considered to be the basic principle of geomechanics, but its appearance makes the task of searching for the stress-strain state as difficult as possible.

For the research objectives, a basic finite-element model of the “horizontal working – rock massif” system is developed, which consists of rectangular and square finite elements and maximally reflects the geometry of the working (**Fig. 1**).

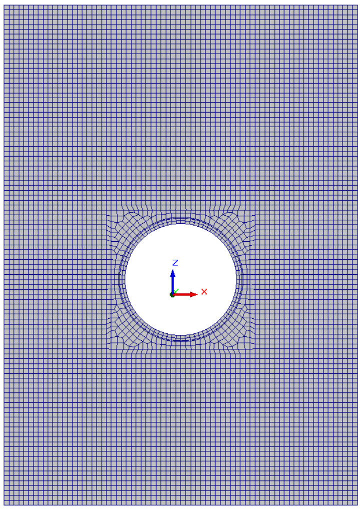


Figure 1. *The finite-element model for the "horizontal working – rock massif" system*

The authors will use the developed models with unit parameters for the primary parametric analysis, implemented with the professional complex SCAD. In these models, the geometric parameters of the underground working are accurately reflected, and the modulus of elasticity E and the density of the rock or soil γ are equal to one. It should also be noted that all further solutions are implemented in an elastic approach since the elastic-plastic solution is quite difficult to solve [12].

The finite element model is a spatial one based on volumetric finite elements. The number of nodes of the model is 5268 pieces, finite elements are 3288 pieces. Model dimensions: height is 20 m, width is 22 m, thickness is 1 m. All nodes of the model are common, and the dimension of finite elements is from 0.25×0.25 to 0.5×0.5 m, which for a model with such dimensions is sufficient to obtain sufficiently accurate results.

Since the stress-strain state of the system changes dramatically while installing fasteners into a horizontal working, it is impossible to conduct a primary parametric analysis of the stress-strain state in a similar way as for an unsupported working. For supported working, a secondary parametric analysis should be conducted proceeding from other initial parameters. This is explained by the fact that the frame is made of different materials (concrete, reinforced concrete, cast iron) and has a thickness range (for horizontal workings with a diameter of 5.6 … 6 m, a thickness of h=0.15 … 0.3 m). Therefore, any change in the parameters of the frame reshapes the stress-strain state in the "horizontal working – rock massif" system, and there are still no final analytical solutions for the distribution of stress and strain components for all possible cases.

In this research, the authors introduce the following proposal, which allows for evaluating the deformed state of a horizontal supported working when there is a change in the thickness of the frame and its material. The controlling parameter affecting the change in displacements of the "horizontal supported working – rock massif" system is both the thickness h and width b of the fastening and to a greater extent the bending stiffness EI. This parameter is integral because it binds the deformation property of the frame material (modulus of elasticity E) and its geometric dimensions (moment of inertia $I = bh^3/12$).

As can be seen from the parametric analysis of the "horizontal supported working – rock massif" system, a stressed state is derived, which is explained by objective reasons. Even if the real deformation characteristics of the rock massif (modulus of elasticity E_r and Poisson's ratio μ_r) is accepted as constant for all possible calculation cases, then the change in the geometric parameters of the fastening (thickness h and width b) leads to a significant change in the stress components. This is due precisely to the change in the interaction between the fastening and the rock massif, which adjust to each other, creating an almost unique stress state for calculated cases with slightly varied thickness h.

Thus, if the bending stiffness EI is taken as the controlling deformation parameter, the following calculation situation can be considered. For two frames of a running tunnel with an inner diameter of 5.1 m and made of different materials (reinforced concrete and cast iron), the equality $E_1I_1 = E_2I_2$ must be:

$$E_1 \frac{b_1 h_1^3}{12} = E_2 \frac{b_2 h_2^3}{12}, \tag{1}$$

which is simplified if the width b is chosen to be the same for two frames (b =1.0 m).

Accordingly, the task, with the help of a somewhat artificial technique, introduced only to simplify it, returns to the search for an equivalent thickness of the frames. However, it should be emphasized that in the general case where the frame width b may not be the same (1.0 m for cast iron frame and 1.2 m for reinforced concrete blocks), the bending stiffness EI remains the controlling parameter. Having set the modulus of elasticity (for cast iron is E_1=20.1·10^4 MPa and reinforced concrete is E_2=32.5·10^3 MPa) and solved equation (1), Eq.(2) is obtained:

$$h_2 = \sqrt[3]{\frac{12E_1I_1}{E_2}}. \tag{2}$$

Accordingly, if the thickness of the cast iron frame h_1=0.15 m is set, the equivalent thickness of the reinforced concrete frame will be equal to h_2=0.275 m (**Fig. 2**).

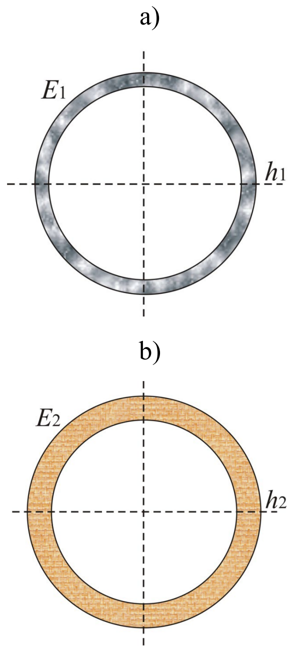


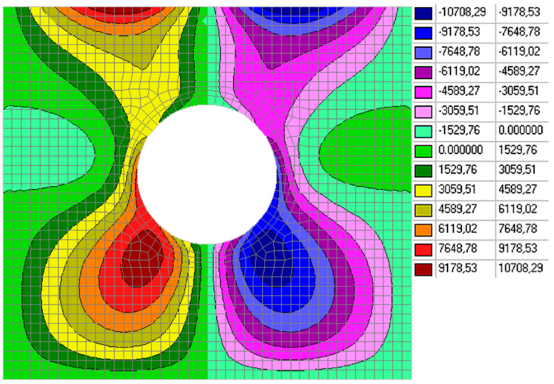
Figure 2. *Schemes of supported horizontal working with an internal diameter of 5.1 m and equivalent EI: a) thickness is of h_1=0.15 m (cast iron); b) thickness is of h_2=0.275 m (reinforced concrete)*

For these two calculation cases, the basic finite-element model (**Fig. 1**) has been modified, since the working is supported. Finite element models are created spatial ones based on volumetric elements. The number of nodes in the model is 18912 (cast iron) and 23156 (reinforced concrete); of finite elements is 9216 (cast iron) and 11316 (reinforced concrete). All nodes in the model are common, the dimension of the finite elements is from 0.12×0.12 m to 0.15×0.18 m. These dimensions are sufficient to obtain an accurate solution to the set task.

III. Results and its analysis

After creating finite-element models for unsupported and supported workings, a numerical analysis of each of them has been carried out. Below are the results on the research of the finite-element model for the unsupported working with unit parameters (**Fig. 3-4**).

a)



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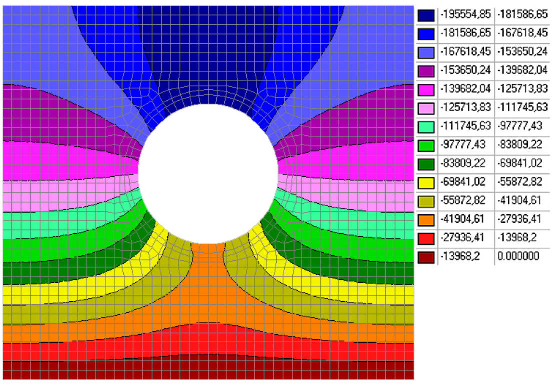
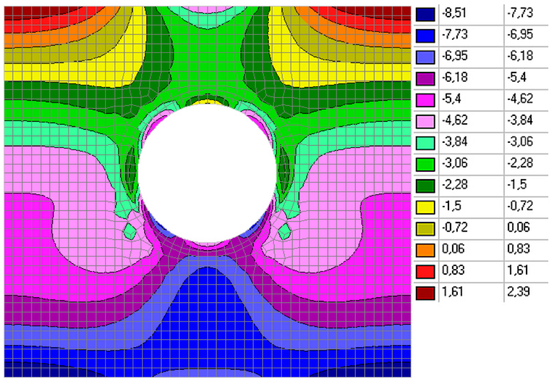


Figure 3. *The strained state of the finite-element model with unit parameters: a) displacement along the horizontal axis; b) displacement along the vertical axis*

Having obtained the distribution of isopoles of stresses and displacements in the model with unit parameters, it is quite easy to move to the real case using the author's algorithm of primary parametric analysis:

a)



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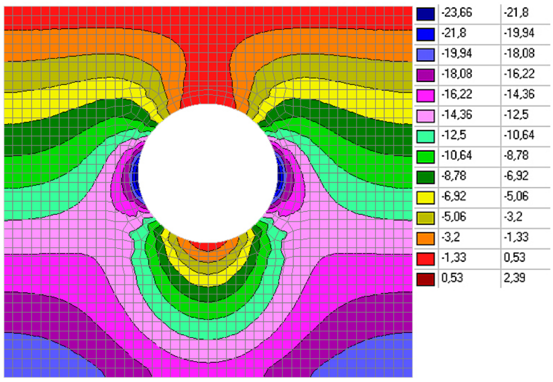


Figure 4. *The stress state of the finite-element model with unit parameters: a) stress along the horizontal axis: b) stress along the vertical axis*

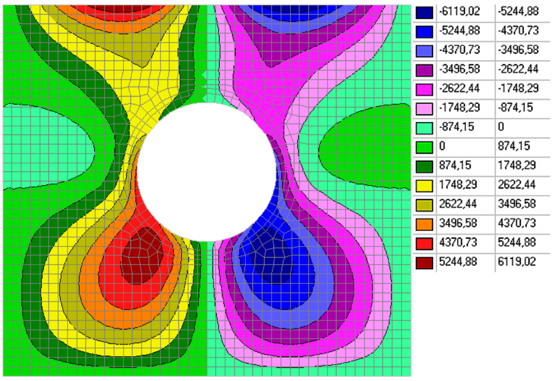
1) to obtain real displacements S_r, the displacement in the model with unit parameters S_e should be multiplied by the value of the real specific gravity γ_r and divided by the real value of the modulus of elasticity E_r and the unit value of the specific gravity:

$$S_r = S_e \frac{\gamma_r}{E_r}; \tag{3}$$

2) to obtain the real stress σ_r , the stress in the model with unit parameters σ_e should be multiplied by the value of the real specific gravity and divided by the unit value of the specific gravity γ_r:

$$\sigma_r = \sigma_e \gamma_r. \tag{4}$$

a)



b)

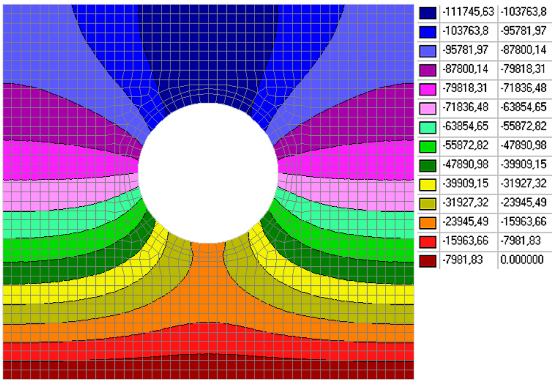


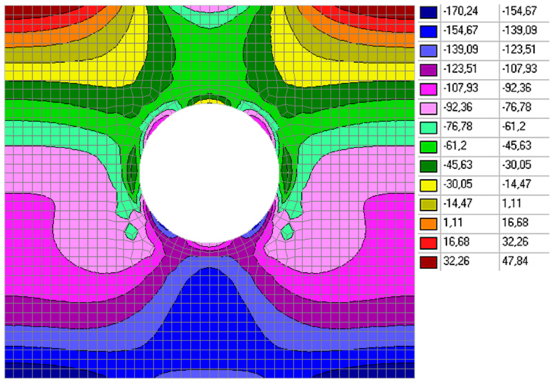
Figure 5. *The strained state of the finite-element model with unit parameters: a) displacement along the horizontal axis; b) displacement along the vertical axis*

To check the adequacy of the distribution of the stress-strain state in the finite-element model and the obtained formulas, below are the results of the calculation of the finite-element model with the values of the real characteristics of the soil (hard clay): modulus of elasticity is of E_r=35 MPa; specific gravity is of γ_r=20 kN/m^3 (**Fig. 5-6**).

As can be seen from the above results of numerical calculations, the isolines and isofields of the stressed and deformed states are qualitatively identical, and they can be obtained quantitatively by the formulas presented above. Thus, if the displacements marked in **Fig. 3**, multiply by 20 (real specific gravity) and divide by 1 (the unit value of specific gravity) and 35 (modulus of elasticity), then the displacement values can be seen in **Fig. 5**. If the stress value in **Fig. 4** multiply by 20 (real specific gravity) and divide by 1 (the unit value of specific gravity), the stress values are received in **Fig. 6**.

In the case of a horizontal supported working, only the strained state is considered, as noted above. To check the adequacy in the distribution of displacements in finite-element models, the calculation results are given below (**Fig. 7-8**).

a)



b)

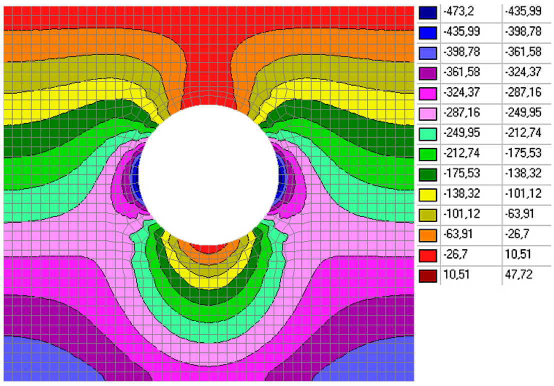


Figure 6. *The stress state of the finite-element model with unit parameters: a) stress along the horizontal axis: b) stress along the vertical axis*

Moreover, the same values of real soil characteristics are reproduced in the models of supported working as for the unsupported one.

The analysis of the components of the strained state (**Fig. 7-8**) makes it possible to conclude that the vertical and horizontal displacements are almost identical for finite-element models with a thickness of h_1=0.15 m (cast iron) and thickness of h_2=0.275 m (reinforced concrete). The error arose due to the fact that the specific gravity, which was considered during the calculation of deformations, for cast iron (72.0 kN/m^3) is almost three times more than for reinforced concrete (24.5 kN/m^3). In the case of horizontal displacements, there was also an error made by the difference between the values of Poisson's ratio (0.3 was for cast iron and 0.2 was for reinforced concrete). It is evidenced by the qualitative distribution of the horizontal component of displacements, since in the case of cast iron, the isofields are more elongated in height, in contrast to the reinforced concrete frame.

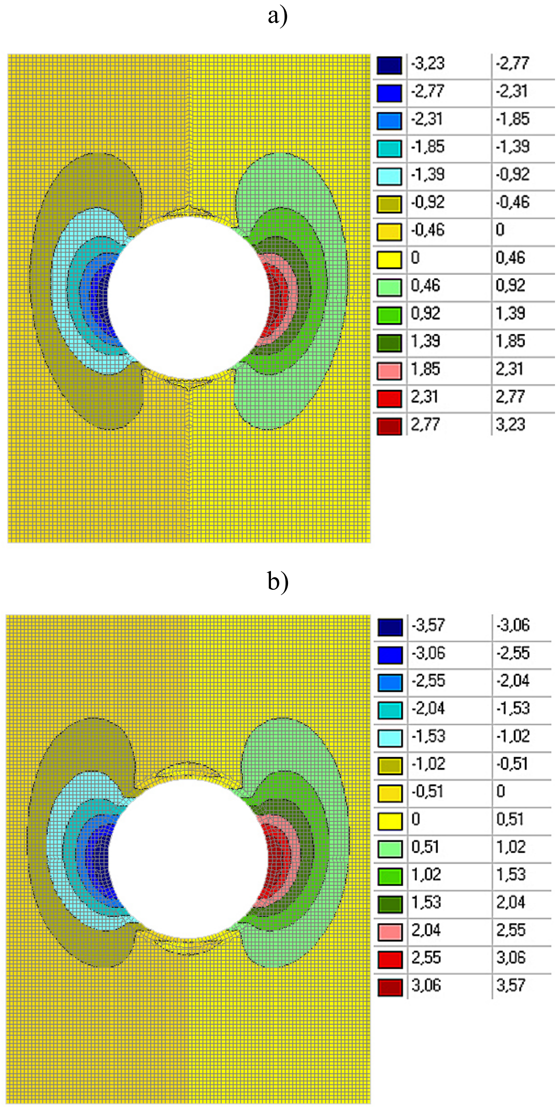


Figure 7. *The strained state of finite-element models (horizontal displacements): a) a cast iron frame with thickness of h_1=0.15 m; b) a reinforced concrete frame with thickness of h_2=0.275 m*

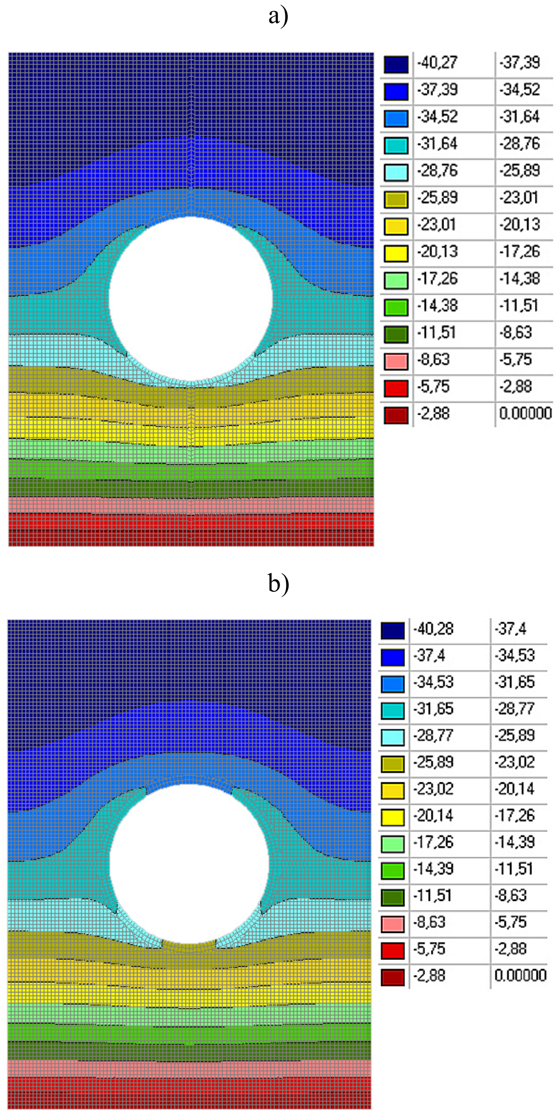


Figure 8. *The strained state of finite-element models (vertical displacements): a) a cast iron frame with thickness of h_1=0.15 m; b) a reinforced concrete frame with thickness of h_2=0.275 m*

The quantitative analysis of the values shows that for the finite-element model with a thickness of h_1=0.15 m (cast iron), the maximum horizontal displacements are 3.23 mm, the maximum vertical displacements are -40.27 mm, and for the finite-element model with a thickness of h_2=0.275 m (reinforced concrete) 3.57 mm and 40.28 mm, respectively (**Fig. 7-8**). The error values (9.5% for horizontal movements and 0.03% for vertical movements) are not significant, and it can be stated that the author's hypothesis about the equality of bending stiffness is fully confirmed.

After this confirmation, a new hypothesis can be introduced that if the equality $E_1 I_1 = E_2 I_2$ exists, then the expression $E_1 I_1 = E_2 I_2 = E_e I_e$ has to also be existed, where E_e and I_e – respectively, some values of the modulus of elasticity and the moment of inertia are equivalent and do not tie to specific materials and sections. This hypothesis makes it possible to create a finite-element model with unit parameters (a thickness is of h_e=0.1 m, and the width is of b_e=1.0 m) of the frame and, by choosing the equivalent modulus of elasticity E_e, to obtain a strained state, which is identical, for example, for the cast iron tubing frame having an open box section. For this, equation (1) should be solved accordingly for the equivalent modulus of elasticity:

$$E_e = \frac{12 \; E_1 I_1}{h_e}, \tag{5}$$

$$E_e = \frac{12 \; E_2 I_2}{h_e}. \tag{6}$$

The resulting formulas implement the search for the equivalent modulus of elasticity E_e both based on the modulus of elasticity for the cast iron E_1, and based on the modulus of elasticity for the reinforced concrete E_2.

IV. Conclusions

The following provisions are conclusions of the performed research of the stress-strain state for unsupported and supported horizontal underground workings.

1. Analysis of the research directions for the stress-strain state of horizontal workings based on the analytical approach and numerical methods, in particular the finite element method, demonstrates that some methodological techniques of the analytical approach can be applied during numerical analysis. After rethinking the analytical approach, the conceptual approach of Kolosov-Mushkelishvili's theory was applied to the author's constructions of parametric analysis for models with unit parameters.

2. The developed system of the parametric analysis proved that to determine the stress-strain state of horizontal unsupported workings of a certain diameter, it is enough to perform one numerical calculation of a finite-element model with unit parameters, and using simple analytical formulas, extrapolate the stress-strain state of this model to all possible cases of real workings.

3. A method of parametric analysis of the strained state of a supported working by using models with equivalent bending stiffness has been developed and substantiated. On its basis, it is possible to perform a numerical analysis of a finite-element model with unit parameters of the frame, selecting the equivalent modulus of elasticity. Thus, the results of the strained state of this model can be extrapolated to models with other geometric parameters of the frame and the material's modulus of elasticity.

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Author Contributions

O. Tiutkin: Conceptualization, Supervision, Theoretical analysis, Writing, Editing.

N. Bondarenko: Finite element modelling, Theoretical analysis, Review, Writing.

Disclosure Statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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