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Determination of the type and the length for the transition curves on the directions of high-speed train operation

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Abstract: In many countries around the world, a cubic parabola is used as transition curves in a simplified form. Today, the properties of many mathematical curves are well studied, which can be used as transition curves. The object of research is the process of restructuring transition curves when organizing high-speed trains operation. The use of modern means of mathematical modelling allows to add a number of dynamic characteristics for evaluation parameters of motion in the curved sections of the tracks. Thus, with an increase in speed for transition curves, both the values of unbalanced acceleration and its change in time, and also the velocity attenuation of oscillations of rolling stock when moving from a transition curve to a circular curve or to a direct section. On the basis of the conducted research, it has been found that when increasing the speed of the vehicle movement in the range from 120 to 200 km/h, the length of the stabilization section increases at the output of the curvature by an average of 20 m for every 40 km/h. The analysis showed that with correctly set parameters of transition curves, which correspond to the maximum speed of movement and radius of a circular curve, the strength and reliability of the bearing elements in the structure of the permanent way and the travel comfort are fully ensured.

Keywords: High-speed train; Railway; Railway curve; Transition curves

I. INTRODUCTION

Before the beginning of the twentieth-century transition curves on the railways in some countries were not foreseen, although then the maximum speeds reached 100 versts per hour (107 km/h), and only since 1912, the arrangement of transition curves has become one of the main requirements. Transition curves were arranged at a radius of 500 sazhens (1067 m) and less. The speed for calculation was taken 60 versts per hour (64 km/h), the transition curve parameter was determined as $C = Rl = 12300$ m^2.

Since the middle of the twentieth century, transition curves should be arranged at a radius of less than 2000 m. In subsequent normative documents of most European countries, transition curves were begun to apply on all curves, radius up to 4000 m inclusive.

In many countries around the world, a cubic parabola is used as transition curves in a simplified form. The transition of such types does not meet all the established requirements [1, 2].

Today, the properties of many mathematical curves are well studied, which can be used as transition curves: the clothoide (also known as Euler spiral or Cornu's spiral), Bernoulli's lemniscate, a cubic parabola, quartic parabola, a high-speed curve, a majorant curve, a minorant curve, etc.

Here are the most common of them with the corresponding curvature equation in **Table 1**.

Thus, in work [3], an overview of the current state of transient curves is given. It is shown that adopted rational parameters of transition curves facilitate the safe and comfortable motion of vehicles for transportation of passengers and cargoes, leading to a decrease in the wear of the permanent way and wheels of rolling stock, which is economically in terms of maintenance. The article defines the objectives of the study related to the geometric continuity of the curve and the dynamics of a transport vehicle for ensuring the smoothness of movement.

It is known that the use of clothoide as a transition curve has become widely applied in many countries of the world. However, a linear curvature is not a sufficient parameter to provide all the requirements of high-speed train traffic. The main results presented in the article [4] are related to the use of new types of transition curves, which turned out to be smoother than clothoide. The proposed method allows to improve the geometry of the existing railways without the need to introduce significant changes in the restructuring of the railway track. In order to eliminate the problems associated with the dynamics of vehicles traffic at high speeds along the transition curves in the form of clothoide, the article [5] analyzes the possibility in the use of the sinusoid as transition curves, which is a diagram of the ideal curvature.

Table 1. *Types of transition curves*

Title	*Equation*
High-speed curve	$k = \cos\beta \sqrt{\frac{2\beta}{C}}$
Quartic parabola	$k = \sqrt[3]{\frac{9}{C} tg^{2/3}\beta \cos^3\beta}$
Majorant curve	$k = 2\sqrt{\frac{tg\beta/2}{C}}$
Minorant curve	$k = \sqrt{\frac{2\sin\beta}{C}}$
Bernoulli's lemniscate	$k = \frac{1}{\sqrt{C}} \frac{\cos^{2/3}\beta}{\sqrt{3\sin^{2/3}\beta}}$
Elastic line	$k = \frac{\sqrt{2C\sin\beta}}{C}$
Sinusoidal curve	$k = \frac{2\pi l - L\sin\left(\frac{2\pi l}{L}\right)}{2\pi LR}$
Cosinusoidal curve	$k = \frac{1 - \cos\left(\frac{\pi l}{L}\right)}{2R}$
Half-sinusoidal curve	$k = \frac{\frac{1}{2R}[1-\cos\beta]}{\left[1+\left(\frac{l}{2\pi R}[\beta - \sin\beta]\right)^2\right]^{\frac{3}{2}}}$
Bloss's curve	$k = \frac{3l^2}{RL^2} - \frac{2l^3}{RL^3}$
Dunin's curve	$k = \frac{1}{R}(3\sigma^2 - 2\sigma^3)$
Shakhuniants's curve	$k = \frac{1}{R}\left(\sigma - \frac{\sin 2\pi\sigma}{2\pi}\right)$
Minorskyi's curve	$k = \frac{8 - 9\cos\pi\sigma + \cos 3\pi\sigma}{16R}$
Koziichuk's curve	$k = \frac{1}{R}\sin^2\frac{\pi\sigma}{2}$
Lokhtman and Rote curve	$k = \frac{1}{R}(10\sigma^3 - 15\sigma^4 + 6\sigma^5)$

In the table: R – radius of the circular curve; l and L – respectively current and full length of the transition curve; $\sigma = \frac{l}{L}$; k –the curvature; β – angle of rotation; $C = Rl$.

Also, there is a transition curve "Wiener Bogen" considering the center of gravity of the vehicle [6, 7]

$$k = k_1 + (k_2 - k_1)f(l) - h(\psi_2 - \psi_1)\frac{d^2 f}{dl^2}$$

where k_1 and k_2 are the curvature at each end, ψ_1 and ψ_2 are the cant values at each end, h is the height to the center of gravity and $f(l)$ is the cant shaping function (chosen among 6 types, of which the most common is a seventh degree polynomial) [8].

In works [9, 10] there are difficulties upon the practical realization and further maintenance of very small horizontal ordinates of the transition curve on the initial section. A new form of a transitional curve is proposed, which, unlike the conventional clothoide, is characterized by a gentle bend in the zone of entry into a circular curve. The apparent advantage of this curve from the viewpoint of its implementation is shown compared to Bloss's graduated transition curves.

In the article [11], the results of optimization of curves are presented through the use of polynomials of the 9th and 11th degrees. The authors use a model of a two-axle rail vehicle in combination with mathematical methods of optimization. The assessment criterion is passengers' comfort. The results obtained for curves, the length of which exceeds 150 m, show that the curve used in the form of the parabola of the 3rd degree is not always optimal. In such cases, the transition curves of the 9th and 11th degrees will be the rational decision.

The article [12] describes the analysis of the results of transitional curves continuation, depending on the track control and modernization of railways. The analysis is based on numerical calculations of a wide range of parameters describing a typical geometric scheme of the railway with transition curves. The radius impact and angle of rotation of the curve on the process of transition curves continuation are considered. On the basis of theoretical assumptions, an effective numerical algorithm of comparative analysis for options of transition curves continuation was developed.

The theoretical foundations in the choice of a rational form of the transition curve developed by Academician V. A. Lazarian are an example of a strict statement of the problem in determining the form of the transition curve, which connects two sections of different curvature, which have a common tangent in the connection point [13].

The authors imply a transitional curve with a limited second derivative of the curvature as the rational form, for which the requirements for angular acceleration, unbalanced acceleration, its changes and steepness of right-of-way for the elevation of an outer rail.

Later, the analytical expressions of the transition curve curvature of the rational form were obtained by V. V. Lahuta under Professor A. A. Bosov [14].

Based on the analysis of graphs for the transition curves curvature in [13], it is possible to conclude that some of them, for example, the curve of V. A. Lazarian, quartic parabola and some others are focused on performing dynamic requirements for angular acceleration in the initial and end points of the transition curve. This condition is quite important for railways, especially for high-speed networks.

Papers [15, 16] deals with the comparative of transition curves used in Hungary are determined. In these works approves that the usability of the clothoide in the Hungarian regulations is unnecessarily restricted for the speeds 120 km/h.

Today, various options of transition curves offered by different scientists are used.

Clothoide, in view of the linear character of a graph of its curvature ($k = \frac{x}{C}$) (**Fig. 1**), is the shortest transition curve, which ensures continuity of the curvature function with the minimum possible rate of change of the curvature for this length ($\frac{dl}{dk}$), and, accordingly, with a minimum rate of growth of centrifugal force at a constant velocity of movement on it.

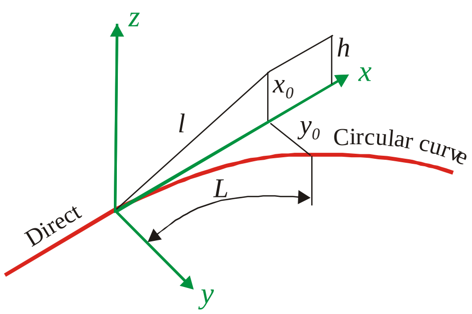

***Figure 1.** Image of the transition curve in two planes*

All other curves, equal to clothoide by the extreme value of the rate of curvature change for these curves, will either be longer clothoide, or have sections with greater than in a clothoide rate of curvature change and, accordingly, with a higher rate of growth of centrifugal for the same constant velocity of motion, that along the clothoide.

It should be noted that the practical application of new types of transition curves, which have a number of excellent properties compared to clothoide, is complicated due to the unresolved issues of normalization of their parameters and support of the horizontal track geometry [17, 18]. Thus, the normalization of the lengths of transition curves depending on the radius that is included in the state-building norms of Ukraine SBN B.2.3-19:2018 [19], can only be applied to clothoide, and only in cases of a connection of direct with circular curves. The normalization of only length is clearly not enough both for the same clothoide, and other transition curves with a nonlinear nature of the change in the curvature. In the publication [20] upon this problem, the authors substantiated the general method of normalizing parameters of any type of transition curves, based on the analysis of the intensity of changing their curvature. This method can be used to clothoide because it does not contradict the theoretical bases of their normalization by parameter $A = \sqrt{C}$. Clothoide parameter C is the magnitude of the inversely proportional rate of changing its curvature over the length, that is $\frac{dl}{dk} = \frac{l}{C} = \frac{l}{RL}$.

Completing the brief overview in the developmental history of the theory of transition curves and the practice of their application, it should be noted that transition curves can be used with regard to the regulatory requirements of safety and smoothness of movement. The very latter circumstance requires a scrupulous analysis in the application of the transition curve in each particular case.

II. Methods

The object of research is the process of restructuring transition curves when organizing high-speed trains.

The transition curve is a spatial curve, in which the curvature both in-plane and in profile smoothly changes (**Fig. 1**). In order to simplify the calculations and break the transition curves, the spatial curve is replaced by a curve of variable curvature only in-plane (**Fig. 2**).

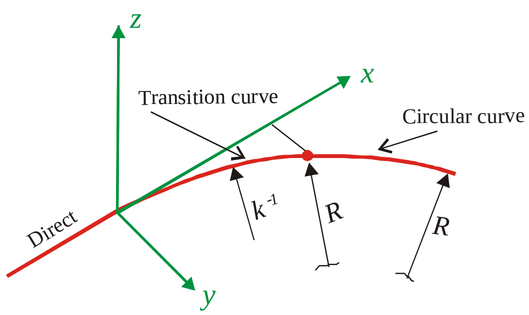

***Figure 2.** Connection of a transition curve with a circular*

As a rule, the transition curve used on the railways of most countries is designed with a linear change of both a radius and elevation of the outer rail. A right-of-way of elevation is determined by the speed of lifting the wheel on it, and the right-of-way of the curvature by the rise speed of a transverse unbalanced acceleration. Such a right-of-way (**Fig. 1**) is easy to arrange and maintain, so it is very common on the railways of many countries. But the transitions of these types do not meet the requirements:

$$\frac{dk}{dL} = 0; \frac{dh}{dL} = 0; \frac{d^2k}{dL^2} = 0; \frac{d^2h}{dL^2} = 0, \qquad (1)$$

where k and h – accordingly, the curvature and elevation of outer rail at the beginning and at the end of the transition curve.

So, for a cubic parabola at the point of connection with a circular curve, we have

$$\frac{d^2k}{dL^2} = \frac{2}{dL^2} \neq 0. \qquad (2)$$

However, neglecting conditions (1) at the beginning and at the end of the transition curve should be compensated by the proper choice of the length for the transition curve to limit the emerging forces and accelerations.

In this paper, creating a new type of transition curves was not a task. Analysis of existing types of transition curves proposed by Shakhuniants, Minorskyi, Dunin, and other scientists (**Fig. 3**) shows that all of them considerably in the initial part has small ordinates and it is difficult to keep them in the correct position on existing lines with the modern design of the upper structure of the track [1].

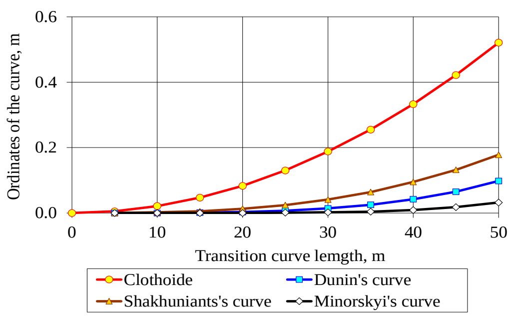

Figure 3. *Changing ordinates along the curve for different types of transition curves*

It is expedient to choose the type of transition curve, that is, function $k = f(l)$, associated with the study of dynamic conditions in the vehicle movement. The analysis showed that sinusoidal curves have good dynamic characteristics for velocities more than 200 km/h [5].

In each case, the type of curve, its length and parameters depend on local relief and situational features and determining conditions of movement.

There is a correct question, whether it is necessary to perform the restructuring of transition curves in order to elongate them, which would meet all necessary requirements at sections of transport corridors, where modernization of the track and artificial structures are carried out in connection with the introduction of high-speed trains movement.

III. MODELING THE PASSENGER CAR MOVEMENT IN THE CURVE OF COMPLEX OUTLINE

Currently, experimental trips with a speed of more than 200 km/year in Ukraine are impossible due to the lack of appropriate rolling stock, and therefore the identification of parameters in designing the plan is based on the modeling the movement of a high-speed car on the curves of a real outlines.

Many works of famous authors are devoted to mathematical modeling of the oscillations of rail vehicles. Such models of rolling stock and track interaction are a system of differential Lagrange equations of the second kind, which describe linear and angular oscillations of the bodies system. As a rule, a local coordinate system is used, that is, a car (locomotive) seems to fluctuate in the same spot under the impact of external forces that show movement. This simplifies some mathematical calculations and increases the accuracy of solving equations (numerical values of oscillations of all bodies are small and approximately the same in all directions). To simulate the movement in the curve, a centrifugal force is applied to the system, which is calculated analytically, based on the curvature of the track under the car. If the curvature is described by complex dependencies or set deterministically with regard to the real outline of the track, the analytically determined definition of centrifugal force is impossible. In such cases, a global coordinate system is used, which allows you to directly specify the coordinate outline of the track and receive the results of centrifugal forces actions as a consequence of the wheels trajectory change which are directed by the positions of rails.

As the basic, the authors accept a model of spatial oscillations of a high-speed passenger car, which was adapted to solve the set problem, including with the implementation in the transition to the global coordinate system [1].

The use of modern means of mathematical modeling allows to add a number of dynamic characteristics for evaluation parameters of motion in the curved sections of the tracks. Thus, with an increase in speed for transition curves, both the values of unbalanced acceleration and its change in time, and also the velocity attenuation of oscillations of rolling stock (updating of a steady motion) when moving from a transition curve to a circular curve or to a direct section. Examples of lateral oscillations damping for the body of the passenger car in the output of the transition curve are shown in **Fig. 4**.

On the basis of the conducted research, it has been found that when increasing the speed of the vehicle

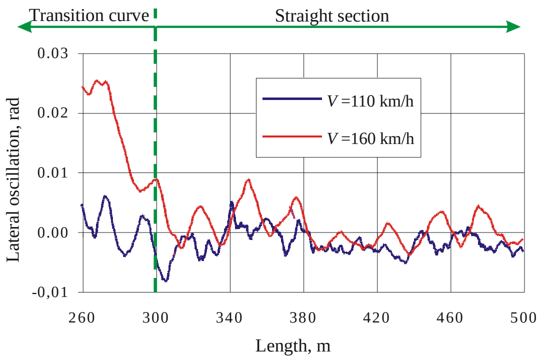

Figure 4. *Lateral oscillation of the car body during the movement along the transition curve*

movement in the range from 120 to 200 km/h, the length of the stabilization section increases at the output of the curvature by an average of 20 m for every 40 km/h.

IV. Determination of the transition curves length in the directions of the introduction of high-speed trains operation

In order to prevent operational and technical problems associated with the possibility of disorders of the permanent way, violations of smoothness and travel comfort in the implementation of the accelerated train movement, the specialization of routes of passenger and freight traffic, taking into account domestic and international needs in transportation is expedient.

Under the conditions of stability, strength and reliability in operation of the track and travel comfort, the structure of the train flow and distribution in it, the categories of trains should be appointed depending on the classification of directions of trains movement [21, 22]. **Table 2** shows characteristics for the direction of high-speed trains traffic. The recommended values of the unbalanced acceleration and its change per unit time are given in **Table 3**.

Table 2. *Characteristics of the direction by categories of trains and the structure of the train flow*

Direction characteristic	***Maximum speed, km / h***	***Categories of trains and the structure of the train flow***	***Maximum load on the axis, kN / axis***
High-speed passenger traffic	$161 < V_{\max pass} \leq 200$	- high-speed passenger trains (85-75%); - accelerated passenger trains (10-15%); - accelerated suburban train (5-10%).	- locomotives of high-speed passenger trains 215; - cars of accelerated passenger trains 180/215; - cars of suburban trains 160.

Table 3. *Security criteria, smoothness and travel comfort*

Direction characteristic	***Maximum permissible unbalanced acceleration α, m / s²***	***Maximum permissible change of the unbalanced acceleration per unit time ψ, m / s³***	***Maximum permissible speed of the wheel elevation on the right-of-way of elevation of outer rail f_v, mm / s***	***Maximum permissible steepness of the right-of-way for elevation of outer rail i, ‰***
High-speed passenger traffic	$\alpha = 0.7$	$\psi = 0.4$	$f_v = 28 \ldots 30$	$i = 0.5 \ldots 0.6$

In the zone of transition curves, the speed of trains is limited by the permissible speed of lifting wheels on the right-of-way for elevation of the outer rail f_v, unbalanced acceleration α and speed of its change in time ψ. And the length of the transition curve is established provided that: the right-of-way for elevation of the outer rail, determined by the speed of the wheel elevation on the right-of-way of outer rail f_v by the equation (3) – condition 1; the right-of-way for the curvature determined by the rate of increase of transverse unbalanced acceleration ψ by the equation (4) – condition 2.

$$l = \frac{hV_{max}}{3.6[f_v]} \tag{3}$$

$$l = \frac{\alpha V_{max}}{3.6[\psi]} \tag{4}$$

Regulatory values $[f_v]$ and $[\psi]$ are accepted by **Table 3**.

In order to calculate the length of the transition curve by the equation (3), it is necessary to set the elevation of outer rail. For high-speed purely passenger traffic

$$\mathrm{h} = \frac{S}{g}\left(\frac{V_{max}^2}{3.6^2 R} - \alpha\right) \quad (5)$$

where S – distance between the axes of the rails; g – acceleration of gravity.

By the equation (5), minimum elevations of the outer rail are calculated, h_{min} in curves for maximum movement speed of high-speed passenger train (**Table 4**).

If the minimum elevation of the outer rail h_{min} in curves of large radii (see **Table 4**) is zero, then the length of the transition curve, in any case, is arranged as necessary for the right-of-way of the curvature (equation 4). The greater value of the transition curve of two obtained by equations (3) and (4) is taken as final. The results of calculations for the following analysis are given in **Table 5** and in **Fig. 5**.

Table 4. *Minimum elevation of outer rail in curves for the direction high-speed traffic*

Radius, m	***Elevation of the outer rail in curves (mm) at the maximum movement speed (km / h)***				
	161	***170***	***180***	***190***	***200***
1400	130	145			
1600	100	115	140		
1800	75	90	110	140	
2000	55	70	90	115	140
2200	40	50	70	90	115
2400	30	35	55	75	95
2600	20	25	45	60	80
2800	10	15	30	50	65
3000	0	5	20	35	55

Table 5. *The length of the transition curves at the maximum speed of the passenger train movement and the radius of the circular curve (direction high-speed traffic, $\alpha = 0.7$ m / s^2)*

Radius, m	***Maximum speed of passenger train movement, km / h***				
	161	***170***	***180***	***190***	***200***
1400	170				
1600	120	170			
1800	90	130	180		
2000	70	100	150	200	
2200	50	80	120	160	
2400	50	60	90	130	180
2600	50	50	70	110	150
2800	50	50	60	80	120
3000	50	50	60	70	100

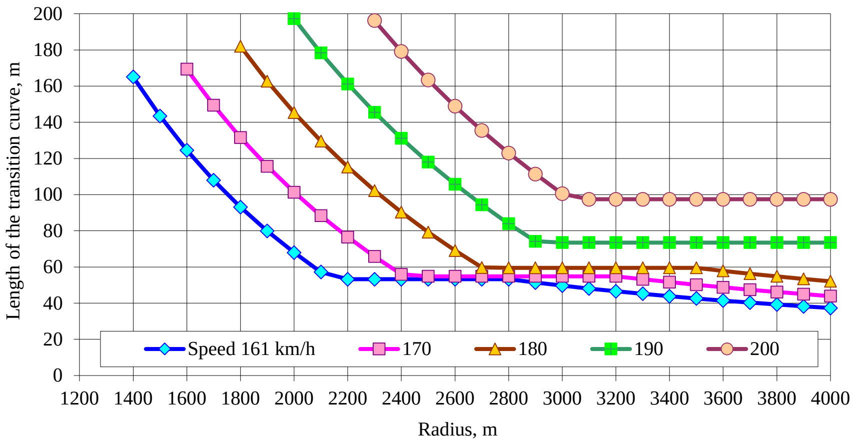

Figure 5. Dependence of the transition curve length from the radius of the curve and the maximum speed of movement in the direction high-speed traffic

The authors offer the unbalanced acceleration to take differentially depending on the distance. This approach allows us to more rationally take into account the need and possibility of restructuring the railway plan when increasing the speed of trains movement. The use of higher values of the unbalanced acceleration makes it possible to realize a higher speed in curves of the average radius (1500-2000 m). For example, at a speed of 161-200 km/h, the required length of the transition curve is reduced by 20-30 m when increasing α from 0.7 m/s^2 to 0.8 m/s^2. When applying α = 0.9 m/s^2 instead of 0.7 m/s^2 the length of the transition curves can be reduced by 40-60 m, and at α = 1.0 m/s^2 the length of the transition curves can be reduced by 50-70 m depending on the maximum speed level.

V. Conclusions

The conducted research showed that the orientation to define the length of the transition curve by the value of the calculated increase, which is determined by the weighted average speed for the train flow, should not be taken on high-speed traffic sections.

The length of the transition curve is determined by the speed of the wheel elevation on the right-of-way of the outer rail f_v (condition 1) and increasing speed of the transverse unbalanced acceleration ψ (condition 2), therefore, there is no clear dependence between the length of the transition curve and the radius of the curve. So, for example:

- at V_{max}=160 km/h the transition curve length l decreases from 170 metres at =1400 m up to 50 metres at R=2200 m (condition 1); in the range R=2200-3300 m, the transition curve length is constant and equal to l=50 m, in the range R=3400-4200 m l=40 m, in the range R=4300-5000 m l=30 m (condition 2);
- at V_{max}=180 km/h the transition curve length l decreases from 180 metres at R=1800 m up to 60 metre at R=2700 m (condition 1); in the range =2700-3700 m the transition curve length is constant and equal to l=60 m, in the range R=3800-4600 m l=50 м , in the range R=4700-5000 m l=40 m (condition 2);
- at V_{max}=200 km/h the transition curve length l decreases from 200 metres at R=2300 m up to 100 metre at R=3000 m (condition 1); in the range R=3000-4500 m the transition curve length is constant and equal l=100 м, in the range R=4500-5000 m l=90 m (condition 2).

The analysis showed that with correctly set parameters of transition curves, which correspond to the maximum speed of movement and radius of a circular curve, the strength and reliability of the bearing elements in the structure of the permanent way and the travel comfort are fully ensured.

Author Contributions

All authors made a substantial, direct, and intellectual contribution to this work. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

Disclosure Statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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