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# Sensitivity Study of a Nonlinear Semi-Active Suspension System

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Abstract: In this paper the OAT (one-at-a-time) sensitivity analysis of a nonlinear semi-active suspension system is carried out with numerical simulation. A specific property of the system is chosen for measure sensitivity, which can be calculated with numerical simulations easily. Both the sensitivity of the system and the input parameters were examined. The degree of sensitivity was measured with a sensitivity index and based on it sensitivity Fuzzy-sets were established. A simple method to reduce sensitivity of a certain parameter is also proposed.

*Keywords: sensitivity analysis, numerical simulation, nonlinear system modelling*

## 1. Introduction

Parameter sensitivity analysis is used in several fields of engineering and science. It is used to find out how the change in parameters affect the systems behaviour [1]. Parameter identification [2] and inverse simulation [3] tasks can also be solved with sensitivity study. From sensitivity, the uncertainty of a system can also be calculated [4]-[5].

In this paper, the local or One-at-a-time (OAT) parameter sensitivity study of a Duffing-type semi-active suspension system is carried out. The aim of this research is to test a simple method to find out which parameters influence the system's behaviour the most and to develop an easy, fast methodology, which can be used in case of more complex systems as well. A specific property of the system is chosen for measure sensitivity, which can be calculated with numerical simulations easily. The presented results can be used later for control tasks.

The paper is organised as follows: after a short literature review the sensitivity analysis with RMS (root mean square) of the acceleration is described, then the examined nonlinear Duffing-type semi-active suspension system is presented shortly. In the next section, the sensitivity analysis of the system and the input parameters is described in detail and a simple method is also proposed to reduce sensitivity.

## 2. Parameter sensitivity analysis

To obtain the sensitivity of a nonlinear system there are several methods ranging from partial differentiation techniques to statistical tests. A detailed review of the most common methods can be found in [6]. As the methods and their applicability are different in this paper only some engineering examples of local sensitivity analysis are presented. A more detailed review of sensitivity study methods and engineering applications is planned in another paper.

Local sensitivity analysis is widely used in environmental engineering tasks. The aim of the study presented in [7] is to identify the most influencing constant parameters of Two-Source Energy Balance Model over an irrigated olive orchard in semi-arid areas with OAT sensitivity study. In [8] OAT sensitivity analysis using Sequential Uncertainty Fitting algorithms was performed to examine the critical input variables of the study area in case of a soil and water assessment tool model for the Langat River basin, Malaysia to predict stream flows. A similar study is presented in [9], where the OAT sensitivity analysis of 13 parameters was carried out to examine the applicability of the SWAT model in the Gumera river basin upstream of Lake Tana, Ethiopia for simulating stream runoff and sediment load. In [10] the local sensitivity analysis of a thermal model of the ZEB Test Cells Laboratory was carried out in terms of temperature profiles of the internal air and internal surfaces with 49 parameters. Based on the local sensitivity analysis and on in-field observations, some actions were suggested to improve the accuracy of the predictions of the thermal behaviour of the test cell.

Local sensitivity studies can be applied in other engineering fields too. In [11] the local sensitivity analysis of a hydraulic servo and a conceptual landing gear model of the Gripen aircraft with Effective Influence Matrix and the Main Sensitivity Index is presented.

Local sensitivity analysis can be effectively used in medical sciences too. It was used for example to find sensitive parameters and their confidence intervals in order to estimate cardiovascular network parameters in the case of the arm arteries [12].

From the literature study, it can be concluded, that OAT sensitivity analysis can be used effectively both in engineering and other sciences as well in order to examine the effects of different parameters on the system's behaviour. In this study, an OAT sensitivity study is carried out to examine the effects of parameter changes in the case of a semi-active suspension system.

### 2.1. Sensitivity analysis of the suspension system

In the case of vibration analysis, the RMS (root mean square) of the acceleration is widely used [13]. It reflects the energy content of the vibration. The RMS of the acceleration can be calculated with the following formula [14]:

$$\mathrm{RMS} = \sqrt{\frac{\sum_{i=1}^{n} a_i^2}{n}} \tag{1}$$

where $a_i$ is the acceleration value at a time interval and $n$ is the number of acceleration values.

The method for calculation of the RMS in numerical simulation can be seen in Fig. 1.

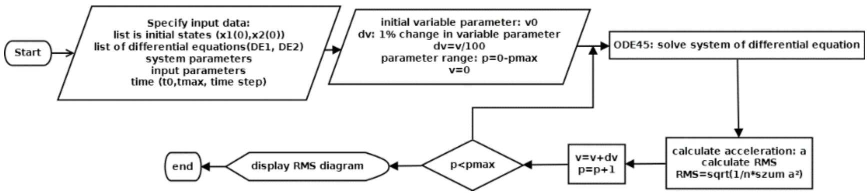


*Figure 1. Calculating the sensitivity diagram*

It was examined how 1% change in the selected parameter changes the RMS value. The parameter range was 0-300%. The parameter is considered sensitive when a small change (1%) in a parameter changes the RMS value rapidly (slope of the RMS curve). Sensitivity is expressed with Sensitivity Index (SI), which can be calculated as the ratio between the relative change in RMS and the relative change in the selected parameter [9]:

$$\mathrm{SI} = \frac{\text{change in RMS } [\%]}{\text{change in parameter } [1\%]} \tag{2}$$

Based on initial simulation results the following Fuzzy sets [15]-[16] are determined depending on the Sensitivity Index to reflect the degree of sensitivity.

1. not sensitive: SI<=0.1

2. moderately sensitive: 0.1<SI<=0.6
3. sensitive: 0.6<SI<2
4. extremely sensitive: 2<SI

In the next sections, the parameter sensitivity analysis of a semi-active suspension system is examined. With the specified Fuzzy-sets it determined how sensitive the system parameters and the input parameters are.

## 3. Semi-active suspension system model

In this study, a semi-active suspension system with magneto-rheological damper is examined with a quarter car model (Fig. 2). This model was previously studied with bifurcation and phase-plane diagrams. It was observed that in extreme cases chaotic oscillation can occur [17].

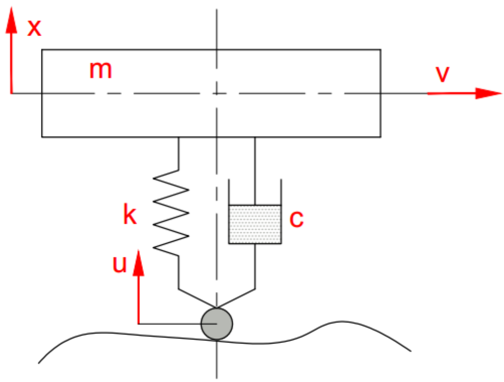


*Figure 2. Semi-active suspension system model*

It is assumed, that the spring has a Duffing type nonlinear restoring force described by the following equation:

$$F_s = -\beta x - \alpha x^3 \tag{3}$$

The damping is assumed to be a variable constant. The road profile was assumed to be sinusoidal, which is the model of an undulated road [19].

The equation describing the system's behaviour is:

$$\frac{d^2}{dt^2}x(t) + \frac{k}{m}\frac{d}{dt}x(t) + \frac{c}{m}\left(-x(t) + 1000x(t)^3\right)x(t) = A\sin\left(\frac{2\pi v}{\lambda}t\right) \tag{4}$$

where *m* is the mass of the vehicle; *k* is the damping coefficient, *c* is the stiffness of the spring, *A* is the amplitude of the road, *v* is the vehicle speed and $\lambda$ is the wavelength of the road. The initial parameters were based on [18] and are shown in Table 1.

*Table 1. Initial simulation parameters*

| ***Parameter*** | ***Name*** | ***Value*** | ***Unit*** |
|---|---|---|---|
| m | mass of vehicle | 375 | kg |
| k | spring stiffness | 35000 | N/m |
| c | damping coefficient | 100 | Ns/m |
| v | vehicle speed | 50 | km/h |
| A | amplitude of road | 0.1 | m |
| λ | wavelength of road | 10 | m |

Numerical simulations were carried out with Maple. *ODE45* numerical solver was used with 0.01 step size and the maximum time was 50 s.

## 4. Sensitivity study of the suspension system

In this section, the sensitivity analysis of the semi-active suspension system is presented. First, the parameters of the suspension, like the mass of the vehicle, the spring stiffness, and the damping coefficient were examined which is followed by the input parameters like the speed of the vehicle and the wavelength and the amplitude of the road. Then a simple method for reducing the sensitivity of an input parameter is proposed.

### 4.1. System parameters

In Fig. 3, the sensitivity of the mass of the vehicle can be seen. When *400 kg<m<600 kg* it is not sensitive. When *500 kg<m<810* kg there is a rapid change in the RMS. *SI=0.42*, therefore this parameter is moderately sensitive. The highest RMS value is at 810 kg, after that it decreases linearly. The RMS value is the lowest when *m=500* kg. If the other parameters won’t change it is advised to enlarge the mass of the vehicle to 400 kg.

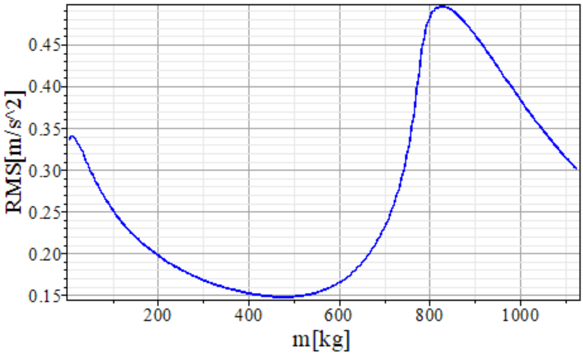


*Figure 3. Sensitivity of the mass*

In Fig. 4, the sensitivity of the damping coefficient can be seen. When the damping coefficient is low ($c<100$ $Ns/m$) it is extremely sensitive, $SI=9$. When $c=0$ chaotic oscillation can also occur [17]. When $100$ $Ns/m<c<500$ $Ns/m$ $SI=0.875$, therefore it is sensitive. When $c>1000$ Ns/m it is not sensitive. It is a good result, as this parameter can be changed, therefore it can be used for control tasks.

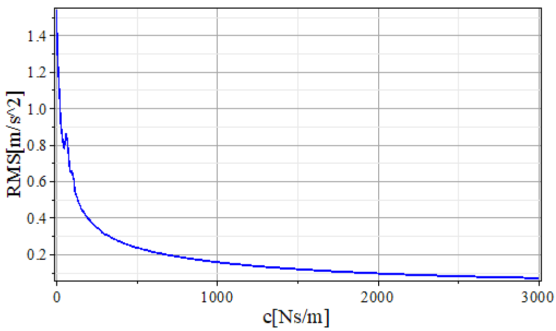


*Figure 4. Sensitivity of the damping coefficient*

In Fig. 5 the sensitivity of the spring stiffness can be seen. It changes the RMS almost linearly, it is not linear only, when it is low ($k<15000$ $N/m$), in that case, the parameter is moderately sensitive with $SI=0.53$. When $k>15000$ $SI=0.175$, which means this parameter is moderately sensitive in this range too.

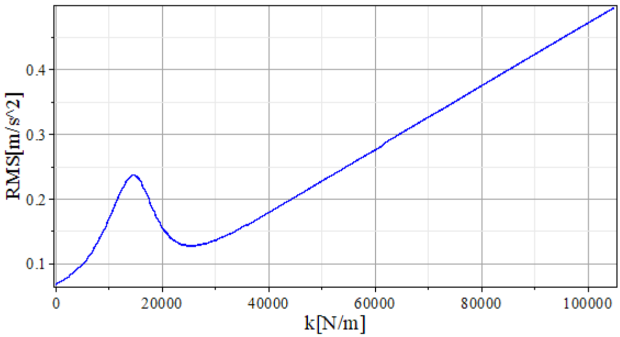


*Figure 5. Sensitivity of the spring stiffness*

### 4.2. Input parameters

In Fig.6 the sensitivity of the vehicle speed can be seen. When $v<60$ *km/h* or $v>110$ *km/h* it is not sensitive. When $60<v<95$ km/h there is a rapid change in the RMS. The highest RMS value is at 77 km/h, therefore it is the most hazardous speed on this road. $SI=0.66$ when $60$ *km/h*$<v<77$ *km/h*, and $SI=0.486$ when $77$*km/h*$<v<95$ *km/h*. This is an important observation as the speed of the vehicle can be easily changed. The sensitive speed range is the case of travelling on a carriageway with a speed limit of 90 km/h.

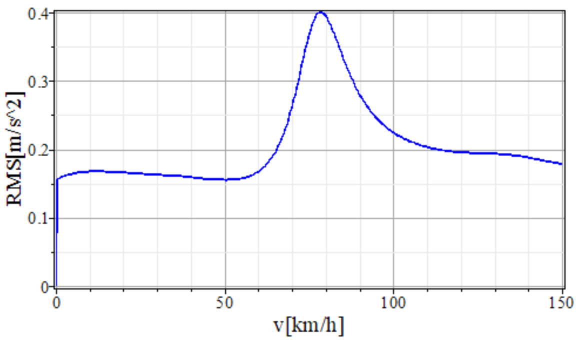


*Figure 6. Sensitivity of the vehicle speed*

In Fig. 7 the sensitivity of the amplitude of the road can be seen. This parameter is not sensitive as the maximum value of $SI=0.03$. The RMS increases linearly as A is increased if $A>0.1$ m.

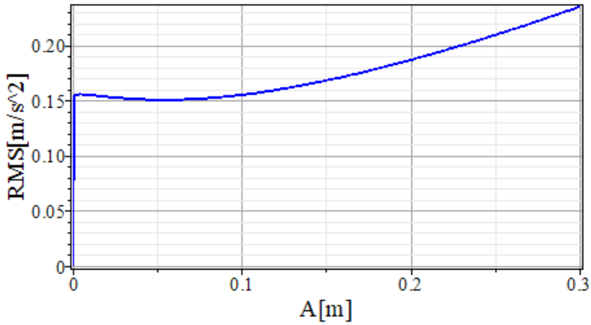


*Figure 7. Sensitivity of the amplitude of the road*

In Fig. 8 the sensitivity of the wavelength of the road can be seen. When $\lambda<3$ *m* or $\lambda>9$ *m* it is not sensitive. When $4$ *m*$<\lambda<6$ *m* SI=1.025 and when $6<\lambda<9$ $SI=0.83$. The wavelength of the road can vary rapidly because of road failures [20], therefore this is the parameter, which should be given particular attention.

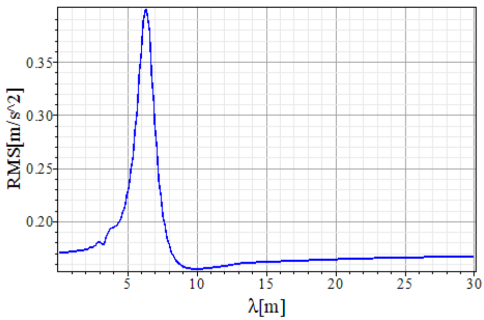


*Figure 8. Sensitivity of wavelength of the road*

### 4.3. Sensitivity of parameters

The results are summarized in Table 2. From the system parameters, the damping coefficient is the most sensitive. It is extremely sensitive when it is of low value. This is the parameter, which can be used for the control task as it can be adjusted. It must be ensured that it should not be reduced below 500 Ns/m and that damper failure not to occur. The mass of the vehicle and the spring stiffness are moderately sensitive and the amplitude of the road is not sensitive. The speed of the vehicle and the wavelength of the road are sensitive. From them, the speed of the vehicle can be controlled. The wavelength of the road can change rapidly and cannot be controlled, therefore its effects are further analysed in the next Session.

*Table 1. Sensitivity of parameters*

| Parameter | $SI_{max}$ | Sensitivity (Fuzzy-set) |
|---|---|---|
| m | 0.42 | 2 |
| c | 9 | 4 |
| k | 0.53 | 2 |
| v | 0.66 | 3 |
| A | 0.03 | 1 |
| λ | 1.25 | 3 |

### 4.4. Reducing sensitivity with adjustable damping coefficient

In this section, it is examined how the sensitivity of the wavelength can be reduced by changing the damping coefficient. The simple case is to enlarge the damping coefficient to until 3000 Ns/m. Another option is to vary the damping coefficient and enlarge it only, where the sensitivity is large and reduce it when it is low. An example of an adjustable damping coefficient can be seen in Fig. 9.

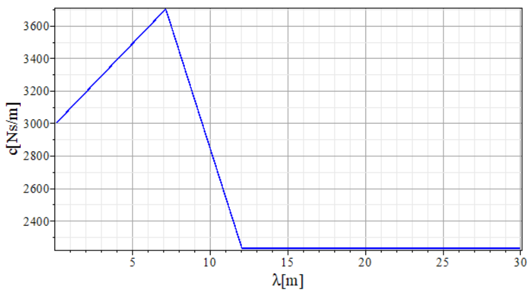


*Figure 9. Adjustable damping coefficient based on the wavelength of the road*

In Fig. 10 the simulation results are shown. It can be seen that with a larger value of damping coefficient not only the sensitivity to the wavelength can be reduced, but the overall value of the RMS too. *SI=0.175* in case of constant *c=3000 Ns/m* and *SI=0.125* in case of adjustable damping coefficient, therefore this parameter becomes moderately sensitive instead of sensitive. It can be concluded that the sensitivity of a parameter can be reduced by changing another parameter. This observation can be important in the case of controller design later on [21].

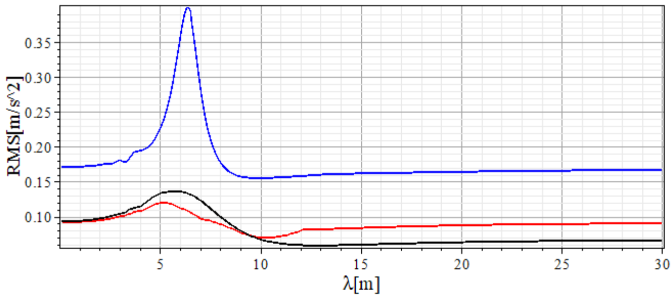


*Figure 10. Sensitivity of the wavelength with different damping coefficients (blue: c=1000, black: c=3000, red: adjustable)*

## 5. Conclusions and further research tasks

In this paper, the OAT sensitivity analysis of a nonlinear semi-active suspension system was carried out with numerical simulations. Fuzzy-sets were established to measure the degree of sensitivity. It was observed that from the system parameters the damping coefficient is the most sensitive, but only when its value is small. From the input parameters, the wavelength of the road is the most sensitive, which can be reduced by changing the damping coefficient. This observation can be the basis of control tasks later. The main task of further research is to examine the effectiveness of the presented method to measure sensitivity and to use it on other examples, like a fire truck suspension system or measured road failures. Later the global sensitivity analysis of the system will be also carried out to acquire more comprehensive knowledge about the behaviour of systems. Another important task is to speed up with parallelization [22]-[24] and to further develop the method in a way to be effectively used in control theory tasks. A long term task is to utilize an optimization algorithm to adjust the parameters to gain an optimal solution in order to reduce the sensitivity of the system.

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