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An Observer-Based *PID* Regulator

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Abstract: An equivalent transfer function representation (*TFR*) is used to study the state-feedback/observer (*SFO*) topologies of control systems. This approach is applied to combine this methodology with YOULA-parametrization (*YP*) introducing new classes of regulators. Then this method is used to introduce observer based *PID* regulators.

Keywords: Observer, state-feedback, model error, YOULA-parametrization, PID regulator

1. Introduction

In our previous paper [6] it was shown that in the classical state-feedback/observer (*SFO*) scheme the model error decreases by the sensitivity function of the observer feedback loop. An equivalent transfer function representation (*TFR*) was used to demonstrate this special feature of these regulators. It was also shown that this principle can be used to generalize for the YOULA-parametrized regulators, too. In this paper the demonstrated new approach is used to introduce further new class of regulators.

2. The Observer Based Youla-Regulator

For open-loop stable processes the all realizable stabilizing (*ARS*) model based regulator $\hat{C}$ is the *YOULA-parametrized* one:

$$\hat{C}\left(\hat{P}\right)=\left.\frac{Q}{1-Q\hat{P}}\right|_{\hat{P}\to P}=\frac{Q}{1-QP}=C(P), \tag{1}$$

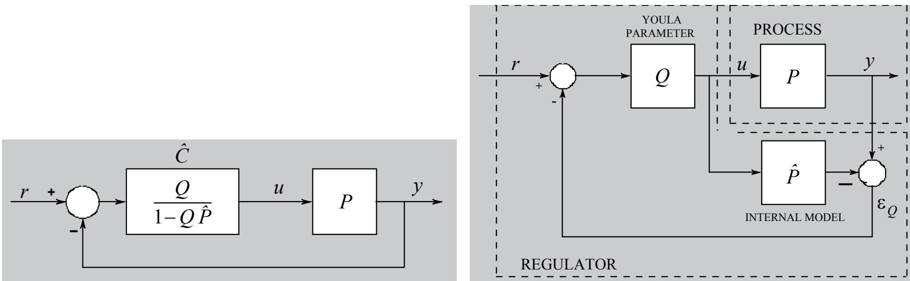
where the "parameter" Q ranges over all proper ($Q(\omega=\infty)$ is finite), stable transfer functions [5], [7], see Fig. 1a.

It is important to know that the *Y-parametrized* closed-loop with the *ARS* regulator is equivalent to the well-known form of the so-called <u>*I*</u>*nternal* <u>*M*</u>*odel* <u>*C*</u>*ontrol* (*IMC*) principle [7] based structure shown in Fig. 1b.

Q is anyway the transfer function from r to u and the closed-loop transfer function (i.e., *CSF*) for $\hat{P} = P$, when $\ell \to 0$

$$\hat{T}_{\mathrm{ry}} = \frac{\hat{C}P}{1+\hat{C}P} = QP \frac{1+\ell}{1+\left(1-QP\right)\ell}\bigg|_{\ell \to 0} = QP = T_{\mathrm{ry}} \tag{2}$$

is linear (and hence convex) in Q.



(a) (b)

Figure 1. The equivalent IMC structure of an ARS regulator

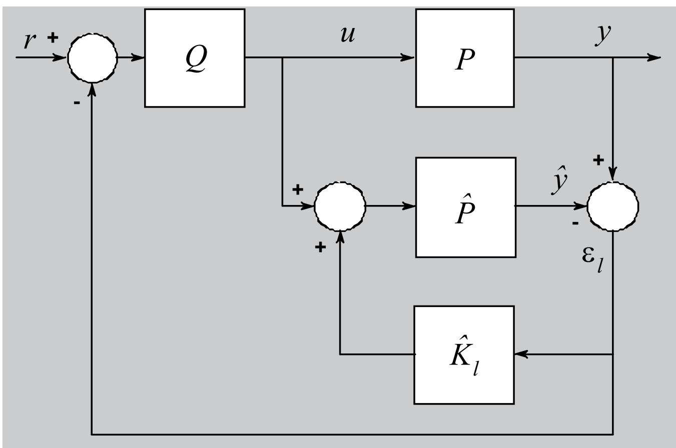


Figure 2. The observer-based IMC structure

It is interesting to compute the relative error ℓ_{T} of $\hat{T}_{\mathrm{ry}}$

$$\ell_{\mathrm{T}} = \frac{T_{\mathrm{ry}} - \hat{T}_{\mathrm{ry}}}{\hat{T}_{\mathrm{ry}}} = \frac{T_{\mathrm{ry}}}{\hat{T}_{\mathrm{ry}}} - 1 = \frac{QP}{\dfrac{QP}{1-Q\left(P-\hat{P}\right)}} - 1 = Q\left(P-\hat{P}\right) = QP\frac{\ell}{1+\ell} = T_{\mathrm{ry}}\frac{\ell}{1+\ell}.$$

(3)

The equivalent *IMC* structure performs the feedback from the model error ε_Q. Similarly to the *SFO* scheme it is possible to construct an internal closed-loop, which virtually reduces the model error to

$$\varepsilon_l = \frac{1}{1+\hat{K}_l\hat{P}}\left(y-\hat{P}u\right) = \frac{1}{1+\hat{K}_l\hat{P}}\varepsilon_Q = \frac{1}{1+\hat{L}_l}\varepsilon_Q = \hat{H}\,\varepsilon_Q\,; \qquad \hat{L}_l = \hat{K}_l\hat{P} \tag{4}$$

and performs the feedback from ε_l (see Fig. 2), where $\hat{L}_l$ is the internal loop transfer function. In this case the resulting closed-loop will change to the scheme shown in Fig. 3.

This means that the introduction of the observer feedback changes the YOULA-*parametrized* regulator to

$$\hat{C}'\left(\hat{P}'\right) = \frac{Q}{1-Q\dfrac{\hat{P}}{1+\hat{K}_l\hat{P}}} = \frac{Q\left(1+\hat{K}_l\hat{P}\right)}{1+\hat{K}_l\hat{P}-Q\hat{P}}\,. \tag{5}$$

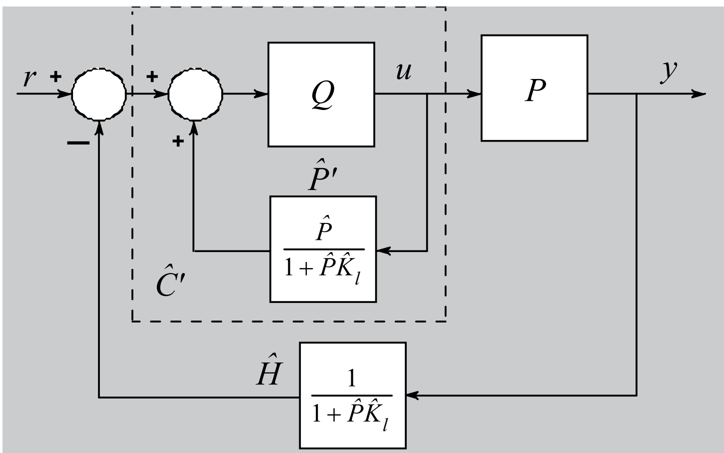


Figure 3. Equivalent closed-loop for the observer-based IMC structure

The form of $\hat{C}'$ shows that the regulator virtually controls a fictitious plant $\hat{P}'$, which is also demonstrated in Fig. 3. Here the fictitious plant is

$$\hat{P}' = \frac{\hat{P}}{1+\hat{K}_l\hat{P}} = \frac{\hat{P}}{1+\hat{L}_l}\,. \tag{6}$$

The closed-loop transfer function is now

$$T'_{\text{ry}} = \frac{\hat{C}'P}{1+\hat{C}'P} = \frac{QP\left(1+\hat{K}_l\hat{P}\right)}{1+\hat{K}_l\hat{P}-Q\hat{P}+QP} = QP\frac{1}{1+QP\dfrac{1}{1+\hat{K}_l\hat{P}}\dfrac{\ell}{1+\ell}}\Bigg|_{\ell\to 0} = QP = T_{\text{ry}} . \tag{7}$$

The relative error ℓ'_{T} of $\hat{T}'_{\text{ry}}$ becomes

$$\ell'_{\text{T}} = \frac{T_{\text{ry}}-\hat{T}'_{\text{ry}}}{\hat{T}'_{\text{ry}}} = \frac{T_{\text{ry}}}{\hat{T}'_{\text{ry}}} - 1 = \frac{QP}{\dfrac{QP\left(1+\hat{K}_l\hat{P}\right)}{1+Q\left(P-\hat{P}\right)+\hat{K}_l\hat{P}}} - 1 = QP\frac{\ell}{1+\ell}\frac{1}{1+\hat{K}_l\hat{P}} = \ell_{\text{T}}\frac{1}{1+\hat{L}_l} \tag{8}$$

which is smaller than ℓ_{T}. The reduction is by $\hat{H} = 1/\left(1+\hat{L}_l\right)$.

3. An Observer Based PID-Regulator

The ideal form of a YOULA-regulator based on reference model design [5] is

$$C_{\text{id}} = \frac{\left(R_{\text{n}}P^{-1}\right)}{1-\left(R_{\text{n}}P^{-1}\right)P} = \frac{Q}{1-QP} = \frac{R_{\text{n}}}{1-R_{\text{n}}}P^{-1} , \tag{9}$$

when the inverse of the process is realizable and stable. Here the operation of R_{n} can be considered a reference model (desired system dynamics). It is generally required that the reference model has to be strictly proper with unit static gain, i.e., $R_{\text{n}}(\omega = 0) = 1$.

For a simple, but robust *PID* regulator design method assume that the process can be well approximated by its two major time constants, i.e.,

$$P \cong \frac{A}{\text{A}_2} , \tag{10}$$

where

$$\text{A}_2 = \left(1+sT_1\right)\left(1+sT_2\right) . \tag{11}$$

According to (9) the ideal YOULA-regulator is

$$C_{\mathrm{id}} = \frac{R_{\mathrm{n}} P^{-1}}{1 - R_{\mathrm{n}}} = \frac{R_{\mathrm{n}} \left(1 + sT_1\right)\left(1 + sT_2\right)}{A\left(1 - R_{\mathrm{n}}\right)}; \quad T_1 > T_2 . \tag{12}$$

Let the reference model R_{n} be of first order

$$R_{\mathrm{n}} = \frac{1}{1 + sT_{\mathrm{n}}} , \tag{13}$$

which means that the first term of the regulator is an integrator

$$\frac{R_{\mathrm{n}}}{1 - R_{\mathrm{n}}} = \frac{\frac{1}{1 + sT_{\mathrm{n}}}}{1 - \frac{1}{1 + sT_{\mathrm{n}}}} = \frac{1}{1 + sT_{\mathrm{n}} - 1} = \frac{1}{sT_{\mathrm{n}}} , \tag{14}$$

whose integrating time is equal to the time constant of the reference model. Thus the resulting regulator corresponds to the design principle, i.e., it is an ideal *PID* regulator

$$C_{\mathrm{PID}} = A_{\mathrm{PID}} \frac{\left(1 + sT_{\mathrm{I}}\right)\left(1 + sT_{\mathrm{D}}\right)}{sT_{\mathrm{I}}} = A_{\mathrm{PID}} \frac{\left(1 + sT_1\right)\left(1 + sT_2\right)}{sT_1} \tag{15}$$

with

$$A_{\mathrm{PID}} = \frac{T_1}{A\,T_{\mathrm{n}}}; \quad T_{\mathrm{I}} = T_1; \quad T_{\mathrm{D}} = T_2 . \tag{16}$$

The YOULA-parameter Q in the ideal regulator is

$$Q = R_{\mathrm{n}} P^{-1} = \frac{1}{A} \frac{\left(1 + sT_1\right)\left(1 + sT_2\right)}{1 + sT_{\mathrm{n}}} . \tag{17}$$

It is not necessary, but desirable to ensure the realizability, i.e., it is reasonable to use

$$Q = R_{\mathrm{n}} P^{-1} = \frac{1}{A} \frac{\left(1 + sT_1\right)\left(1 + sT_2\right)}{\left(1 + sT_{\mathrm{n}}\right)\left(1 + sT\right)} , \tag{18}$$

where T can be considered as the time constant of the derivative action ($0.1 T_{\mathrm{D}} \le T \le 0.5 T_{\mathrm{D}}$). The regulator $\hat{C}'$ and the feedback term $\hat{H}$ must be always realizable. In the practice the *PID* regulator and the YOULA-parameter is always model-based, so

$$\hat{C}_{\mathrm{PID}}\left(\hat{P}\right) = \hat{A}_{\mathrm{PID}} \frac{\left(1 + s\hat{T}_1\right)\left(1 + s\hat{T}_2\right)}{s\hat{T}_1}; \quad \hat{A}_{\mathrm{PID}} = \frac{\hat{T}_1}{\hat{A}\,T_{\mathrm{n}}} , \tag{19}$$

$$\hat{Q} = R_{\mathrm{n}} \hat{P}^{-1} = \frac{1}{\hat{A}} \frac{\left(1 + s\hat{T}_1\right)\left(1 + s\hat{T}_2\right)}{1 + sT_{\mathrm{n}}} . \tag{20}$$

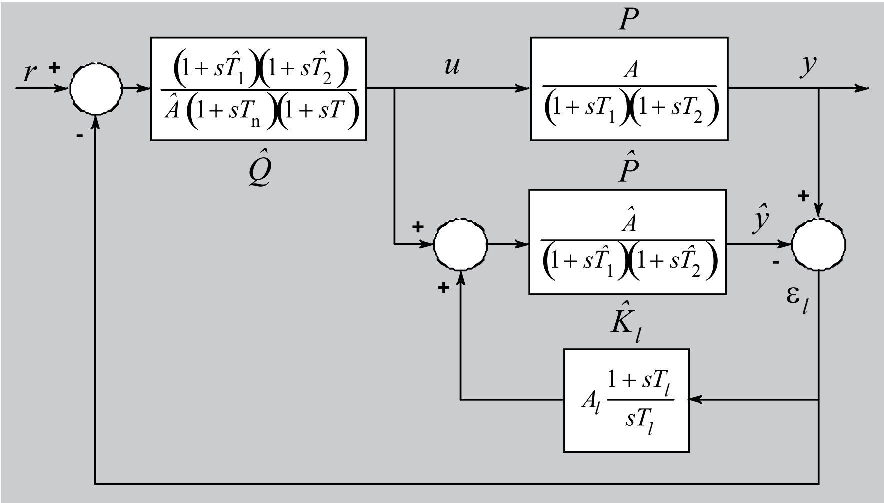


Figure 4. An observer based PID regulator

The scheme of the observer based *PID* regulator is shown in Fig. 4, where a simple *PI* regulator

$$\hat{K}_l = A_l \frac{1+sT_l}{sT_l} \tag{21}$$

is applied in the observer-loop. Here T_l must be in the range of *T*, i.e., considerably smaller than T_1 and T_2.

Note that the frequency characteristic of $\hat{H}$ cannot be easily designed to reach a proper error suppression. For example, it is almost impossible to design a good realizable high cut filter in this architecture. The high frequency domain is always more interesting to speed up a control loop, so the target of the future research is how to select $\hat{K}_l$ for the desired shape of $\hat{H}$.

4. Simulation Experiments

The simulation experiments were performed in using the observer based *PID* scheme shown in Fig. 4.

Example 1

The process parameters are: $T_1 = 20$, $T_2 = 10$ and $A = 1$. The model parameters are: $\hat{T}_1 = 25$, $\hat{T}_2 = 12$ and $\hat{A} = 1.2$. The purpose of the regulation is to speed up the basic step response by 4, i.e., $T_n = 5$ is selected in the first order R_n. In the observer loop a simple proportional regulator $\hat{K}_l = 0.01$ is applied. The ideal form of *Q* (17) was used. Figure 5 shows some step responses in the operation of the observer based *PID* regulator.

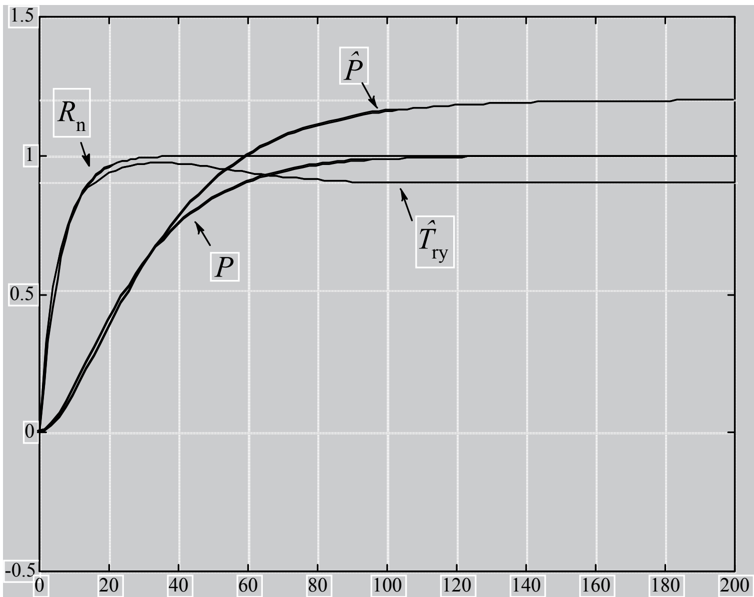


Figure 5. Step responses using the observer based PID regulator

It is easy to see that the $\hat{T}'_{ry}$ very well approximates R_n in the high frequencies (for small time values) in spite of the very bad model $\hat{P}$.

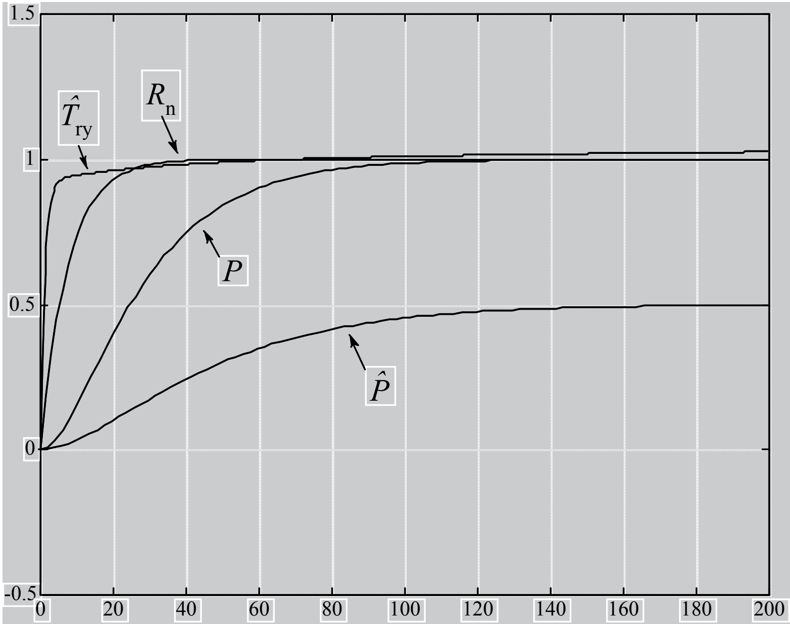


Figure 6. Step responses using the observer based PID regulator

Example 2

The process parameters and the selected first order R_n are the same as in the previous example. The model parameters are: $\hat{T}_1 = 30$, $\hat{T}_2 = 20$ and $\hat{A} = 0.5$. In the observer loop a *PI* regulator (67) is applied with $A_l = 0.001$ and $T_l = 2$. The ideal form of *Q* (17) was used. Figure 6 shows some step responses in the operation of the observer based *PID* regulator.

It is easy to see that the $\hat{T}'_{ry}$ well approximates R_n in the high frequencies (for small time values) in spite of the very bad model $\hat{P}$.

5. Conclusions

It was shown that the *SFO* methodology can be applied to the YOULA-parametrized regulators, too. This approach reduces the model error by the sensitivity function of the observer loop similarly to the *SFO* scheme.

Using this new approach an observer based *PID* regulator was also introduced. This regulator works well even in case of large model errors as some simulations showed.

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